# Algebra I-B B.E.S.T. Instructional Guide for Mathematics

The B.E.S.T. Instructional Guide for Mathematics (B1G-M) is intended to assist educators with planning for student learning and instruction aligned to Florida's Benchmarks for Excellent Student Thinking (B.E.S.T.) Standards. This guide is designed to aid high-quality instruction through the identification of components that support the learning and teaching of the B.E.S.T. Mathematics Standards and Benchmarks. The B1G-M includes an analysis of information related to the B.E.S.T. Standards for Mathematics within this specific mathematics course, the instructional emphasis and aligned resources. This document is posted on the B.E.S.T. Standards for Mathematics webpage of the Florida Department of Education's website and will continue to undergo edits as needed.

## Structural Framework and Intentional Design of the B.E.S.T. Standards for Mathematics

Florida's B.E.S.T. Standards for Mathematics were built on the following.

- The coding scheme for the standards and benchmarks was changed to be consistent with other content areas. The new coding scheme is structured as follows: Content.GradeLevel.Strand.Standard.Benchmark.
- Strands were streamlined to be more consistent throughout.
- The standards and benchmarks were written to be clear and concise to ensure that they are easily understood by all stakeholders.
- The benchmarks were written to allow teachers to meet students' individual skills, knowledge and ability.
- The benchmarks were written to allow students the flexibility to solve problems using a method or strategy that is accurate, generalizable and efficient depending on the content (i.e., the numbers, expressions or equations).
- The benchmarks were written to allow for student discovery (i.e., exploring) of strategies rather than the teaching, naming and assessing of each strategy individually.
- The benchmarks were written to support multiple pathways for success in career and college for students.
- The benchmarks should not be taught in isolation but should be combined purposefully.
- The benchmarks may be addressed at multiple points throughout the year, with the intention of gaining mastery by the end of the year.
- Appropriate progression of content within and across strands was developed for each grade level and across grade levels.
- There is an intentional balance of conceptual understanding and procedural fluency with the application of accurate real-world context intertwined within mathematical concepts for relevance.
- The use of other content areas, like science and the arts, within real-world problems should be accurate, relevant, authentic and reflect grade-level appropriateness.

## Components of the B.E.S.T. Instructional Guide for Mathematics

The following table is an example of the layout for each benchmark and includes the defining attributes for each component. It is important to note that instruction should not be limited to the possible connecting benchmarks, related terms, strategies or examples provided. To do so would strip the intention of an educator meeting students' individual skills, knowledge and abilities.

**Benchmark**
*focal point for instruction within lesson or task*

This section includes the benchmark as identified in the B.E.S.T. Standards for Mathematics. The benchmark, also referred to as the Benchmark of Focus, is the focal point for student learning and instruction. The benchmark, and its related example(s) and clarification(s), can also be found in the course description. The 9-12 benchmarks may be included in multiple courses; select the example(s) or clarification(s) as appropriate for the identified course.

**Connecting Benchmarks/Horizontal Alignment**
*in other standards within the grade level or course*

This section includes a list of connecting benchmarks that relate horizontally to the Benchmark of Focus. Horizontal alignment is the intentional progression of content within a grade level or course linking skills within and across strands. Connecting benchmarks are benchmarks that either make a mathematical connection or include prerequisite skills. The information included in this section is not a comprehensive list, and educators are encouraged to find other connecting benchmarks. Additionally, this list will not include benchmarks from the same standard since benchmarks within the same standard already have an inherent connection.

**Terms from the K-12 Glossary**

This section includes terms from Appendix C: K-12 Glossary, found within the B.E.S.T. Standards for Mathematics document, which are relevant to the identified Benchmark of Focus. The terms included in this section should not be viewed as a comprehensive vocabulary list, but instead should be considered during instruction or act as a reference for educators.

**Vertical Alignment**
*across grade levels or courses*

This section includes a list of related benchmarks that connect vertically to the Benchmark of Focus. Vertical alignment is the intentional progression of content from one year to the next, spanning across multiple grade levels. Benchmarks listed in this section make mathematical connections from prior grade levels or courses in future grade levels or courses within and across strands. If the Benchmark of Focus is a new concept or skill, it may not have any previous benchmarks listed. Likewise, if the Benchmark of Focus is a mathematical skill or concept that is finalized in learning and does not have any direct connection to future grade levels or courses, it may not have any future benchmarks listed. The information included in this section is not a comprehensive list, and educators are encouraged to find other benchmarks within a vertical progression.

**Purpose and Instructional Strategies**

This section includes further narrative for instruction of the benchmark and vertical alignment. Additionally, this section may also include the following:

- explanations and details for the benchmark;
- vocabulary not provided within Appendix C;
- possible instructional strategies and teaching methods; and
- strategies to embed potentially related Mathematical Thinking and Reasoning Standards (MTRs).

### Common Misconceptions or Errors

This section will include common student misconceptions or errors and may include strategies to address the identified misconception or error. Recognition of these misconceptions and errors enables educators to identify them in the classroom and make efforts to correct the misconception or error. This corrective effort in the classroom can also be a formative assessment within instruction.

### Strategies to Support Tiered Instruction

The instructional strategies in this section address the common misconceptions and errors listed within the above section that can be a barrier to successfully learning the benchmark. All instruction and intervention at Tiers 2 and 3 are intended to support students to be successful with Tier 1 instruction. Strategies that support tiered instruction are intended to assist teachers in planning across any tier of support and should not be considered exclusive or inclusive of other instructional strategies that may support student learning with the B.E.S.T. Mathematics Standards. For more information about tiered instruction, please see the Effective Tiered Instruction for Mathematics: ALL Means ALL document.

### Instructional Tasks

*demonstrate the depth of the benchmark and the connection to the related benchmarks*

This section will include example instructional tasks, which may be open-ended and are intended to demonstrate the depth of the benchmark. Some instructional tasks include integration of the Mathematical Thinking and Reasoning Standards (MTRs) and related benchmark(s). Enrichment tasks may be included to make connections to benchmarks in later grade levels or courses. Tasks may require extended time, additional materials and collaboration.

### Instructional Items

*demonstrate the focus of the benchmark*

This section will include example instructional items which may be used as evidence to demonstrate the students' understanding of the benchmark. Items may highlight one or more parts of the benchmark.

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**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

## Mathematical Thinking and Reasoning Standards
*MTRs: Because Math Matters*

Florida students are expected to engage with mathematics through the Mathematical Thinking and Reasoning Standards (MTRs) by utilizing their language as a self-monitoring tool in the classroom, promoting deeper learning and understanding of mathematics. The MTRs are standards which should be used as a lens when planning for student learning and instruction of the B.E.S.T. Standards for Mathematics.

### Structural Framework and Intentional Design of the Mathematical Thinking and Reasoning Standards

The Mathematical Thinking and Reasoning Standards (MTRs) are built on the following.

- The MTRs have the same coding scheme as the standards and benchmarks; however, they are written at the standard level because there are no benchmarks.
- In order to fulfill Florida's unique coding scheme, the 5th place (benchmark) will always be a "1" for the MTRs.
- The B.E.S.T. Standards for Mathematics should be taught through the lens of the MTRs.
- At least one of the MTRs should be authentically and appropriately embedded throughout every lesson based on the expectation of the benchmark(s).
- The bulleted language of the MTRs was written for students to use as self-monitoring tools during daily instruction.
- The clarifications of the MTRs were written for teachers to use as a guide to inform their instructional practices.
- The MTRs ensure that students stay engaged, persevere in tasks, share their thinking, balance conceptual understanding and procedures, assess their solutions, make connections to previous learning and extended knowledge, and apply mathematical concepts to real-world applications.
- The MTRs should not stand alone as a separate focus for instruction but should be combined purposefully.
- The MTRs will be addressed at multiple points throughout the year, with the intention of gaining mastery of mathematical skills by the end of the year and building upon these skills as they continue in their K-12 education.

### MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Mathematicians who participate in effortful learning both individually and with others:

- Analyze the problem in a way that makes sense given the task.
- Ask questions that will help with solving the task.
- Build perseverance by modifying methods as needed while solving a challenging task.
- Stay engaged and maintain a positive mindset when working to solve tasks.
- Help and support each other when attempting a new method or approach.

<u>Clarifications:</u>

Teachers who encourage students to participate actively in effortful learning both individually and with others:

- Cultivate a community of growth mindset learners.
- Foster perseverance in students by choosing tasks that are challenging.
- Develop students' ability to analyze and problem solve.
- Recognize students' effort when solving challenging problems.

## MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Mathematicians who demonstrate understanding by representing problems in multiple ways:

- Build understanding through modeling and using manipulatives.
- Represent solutions to problems in multiple ways using objects, drawings, tables, graphs and equations.
- Progress from modeling problems with objects and drawings to using algorithms and equations.
- Express connections between concepts and representations.
- Choose a representation based on the given context or purpose.

Clarifications:
Teachers who encourage students to demonstrate understanding by representing problems in multiple ways:

- Help students make connections between concepts and representations.
- Provide opportunities for students to use manipulatives when investigating concepts.
- Guide students from concrete to pictorial to abstract representations as understanding progresses.
- Show students that various representations can have different purposes and can be useful in different situations.

## MA.K12.MTR. 3.1 Complete tasks with mathematical fluency.

Mathematicians who complete tasks with mathematical fluency:

- Select efficient and appropriate methods for solving problems within the given context.
- Maintain flexibility and accuracy while performing procedures and mental calculations.
- Complete tasks accurately and with confidence.
- Adapt procedures to apply them to a new context.
- Use feedback to improve efficiency when performing calculations.

Clarifications:
Teachers who encourage students to complete tasks with mathematical fluency:

- Provide students with the flexibility to solve problems by selecting a procedure that allows them to solve them efficiently and accurately.
- Offer multiple opportunities for students to practice efficient and generalizable methods.
- Provide opportunities for students to reflect on the method they used and determine if a more efficient method could have been used.

## MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:

- Communicate mathematical ideas, vocabulary and methods effectively.
- Analyze the mathematical thinking of others.
- Compare the efficiency of a method to those expressed by others.
- Recognize errors and suggest how to correctly solve the task.
- Justify results by explaining methods and processes.
- Construct arguments based on evidence.

Clarifications:
Teachers who encourage students to engage in discussions that reflect on the mathematical thinking of self and others:

- Establish a culture in which students ask questions of the teacher and their peers, and error is an opportunity for learning.
- Create opportunities for students to discuss their thinking with peers.
- Select, sequence and present student work to advance and deepen understanding of correct and increasingly efficient methods.
- Develop students' ability to justify methods and compare their responses to the responses of their peers.

### MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

- Focus on relevant details within a problem.
- Create plans and procedures to logically order events, steps or ideas to solve problems.
- Decompose a complex problem into manageable parts.
- Relate previously learned concepts to new concepts.
- Look for similarities among problems.
- Connect solutions of problems to more complicated large-scale situations.

Clarifications:
Teachers who encourage students to use patterns and structure to help understand and connect mathematical concepts:

- Help students recognize the patterns in the world around them and connect these patterns to mathematical concepts.
- Support students to develop generalizations based on the similarities found among problems.
- Provide opportunities for students to create plans and procedures to solve problems.
- Develop students' ability to construct relationships between their current understanding and more sophisticated ways of thinking.

### MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:

- Estimate to discover possible solutions.

- Use benchmark quantities to determine if a solution makes sense.
- Check calculations when solving problems.
- Verify possible solutions by explaining the methods used.
- Evaluate results based on the given context.

Clarifications:
Teachers who encourage students to assess the reasonableness of solutions:
- Have students estimate or predict solutions prior to solving.
- Prompt students to continually ask, “Does this solution make sense? How do you know?”
- Reinforce that students check their work as they progress within and after a task.
- Strengthen students’ ability to verify solutions through justifications.

## MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:
- Connect mathematical concepts to everyday experiences.
- Use models and methods to understand, represent and solve problems.
- Perform investigations to gather data or determine if a method is appropriate.
- Redesign models and methods to improve accuracy or efficiency.

Clarifications:
Teachers who encourage students to apply mathematics to real-world contexts:
- Provide opportunities for students to create models, both concrete and abstract, and perform investigations.
- Challenge students to question the accuracy of their models and methods.
- Support students as they validate conclusions by comparing them to the given situation.
- Indicate how various concepts can be applied to other disciplines.

## Examples of Teacher and Student Moves for the MTRs

Below are examples that show embedding the MTRs within the mathematics classroom. The provided teacher and student moves are examples of how some MTRs could be incorporated into student learning and instruction. The information included in this table is not a comprehensive list, and educators are encouraged to incorporate other teacher and student moves that support the MTRs.

| MTR | Student Moves | Teacher Moves |
|---|---|---|
| MA.K12.MTR.1.1 *Actively participate in effortful learning both individually and collectively.* | • Students engage in the task through individual analysis, student-to-teacher interaction and student-to-student interaction.<br>• Students ask task-appropriate questions to self, the teacher and to other students. *(MTR.4.1)*<br>• Students have a positive productive struggle exhibiting growth mindset, even when making a mistake.<br>• Students stay engaged in the task to a purposeful conclusion while modifying methods, when necessary, in solving a problem through self-analysis and perseverance. | • Teacher provides flexible options (i.e., differentiated, challenging tasks that allow students to actively pursue a solution both individually and in groups) so that all students have the opportunity to access and engage with instruction, as well as demonstrate their learning.<br>• Teacher creates a physical environment that supports a growth mindset and will ensure positive student engagement and collaboration.<br>• Teacher provides constructive, encouraging feedback to students that recognizes their efforts and the value of analysis and revision.<br>• Teacher provides appropriate time for student processing, productive struggle and reflection.<br>• Teacher uses data and questions to focus students on their thinking; help students determine their sources of struggle and to build understanding.<br>• Teacher encourages students to ask appropriate questions of other students and of the teacher including questions that examine accuracy. *(MTR.4.1)* |

| MTR | Student Moves | Teacher Moves |
|---|---|---|
| MA.K12.MTR.2.1 *Demonstrate understanding by representing problems in multiple ways.* | • Students represent problems concretely using objects, models and manipulatives.<br>• Students represent problems pictorially using drawings, models, tables and graphs.<br>• Students represent problems abstractly using numerical or algebraic expressions and equations.<br>• Students make connections and select among different representations and methods for the same problem, as appropriate to different situations or context. *(MTR.3.1)* | • Teacher provides students with objects, models, manipulatives, appropriate technology and real-world situations. *(MTR.7.1)*<br>• Teacher encourages students to use drawings, models, tables, expressions, equations and graphs to represent problems and solutions.<br>• Teacher questions students about making connections between different representations and methods and challenges students to choose one that is most appropriate to the context. *(MTR.3.1)*<br>• Teacher encourages students to explain their different representations and methods to each other. *(MTR.4.1)*<br>• Teacher provides opportunities for students to choose appropriate methods and to use mathematical technology. |
| MA.K12.MTR.3.1 *Complete tasks with mathematical fluency.* | • Students complete tasks with flexibility, efficiency and accuracy.<br>• Students use feedback from peers and teachers to reflect on and revise methods used.<br>• Students build confidence through practice in a variety of contexts and problems. *(MTR.1.1)* | • Teacher provides tasks and opportunities to explore and share different methods to solve problems. *(MTR.1.1)*<br>• Teacher provides opportunities for students to choose methods and reflect (i.e., through error analysis, revision, summarizing methods or writing) on the efficiency and accuracy of the method(s) chosen.<br>• Teacher asks questions and gives feedback to focus student thinking to build efficiency of accurate methods.<br>• Teacher offers multiple opportunities to practice generalizable methods. |



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| MTR | Student Moves | Teacher Moves |
|---|---|---|
| MA.K12.MTR.4.1 *Engage in discussions that reflect on the mathematical thinking of self and others.* | • Students use content specific language to communicate and justify mathematical ideas and chosen methods.<br>• Students use discussions and reflections to recognize errors and revise their thinking.<br>• Students use discussions to analyze the mathematical thinking of others.<br>• Students identify errors within their own work and then determine possible reasons and potential corrections.<br>• When working in small groups, students recognize errors of their peers and offers suggestions. | • Teacher provides students with opportunities (through open-ended tasks, questions and class structure) to make sense of their thinking. *(MTR.1.1)*<br>• Teacher uses precise mathematical language, both written and abstract, and encourages students to revise their language through discussion.<br>• Teacher creates opportunities for students to discuss and reflect on their choice of methods, their errors and revisions and their justifications.<br>• Teachers select, sequence and present student work to elicit discussion about different methods and representations. *(MTR.2.1, MTR.3.1)* |

| MTR | Student Moves | Teacher Moves |
|---|---|---|
| MA.K12.MTR.5.1<br>*Use patterns and structure to help understand and connect mathematical concepts.* | • Students identify relevant details in a problem to create plans and decompose problems into manageable parts.<br>• Students find similarities and common structures, or patterns, between problems to solve related and more complex problems using prior knowledge. | • Teacher asks questions to help students construct relationships between familiar and unfamiliar problems and to transfer this relationship to solve other problems. *(MTR.1.1)*<br>• Teacher provides students opportunities to connect prior and current understanding to new concepts.<br>• Teacher provides opportunities for students to discuss and develop generalizations about a mathematical concept. *(MTR.3.1, MTR.4.1)*<br>• Teacher allows students to develop an appropriate sequence of steps in solving problems.<br>• Teacher provides opportunities for students to reflect during problem solving to make connections to problems in other contexts, noticing structure and making improvements to their process. |
| MA.K12.MTR.6.1<br>*Assess the reasonableness of solutions.* | • Students estimate a solution, including using benchmark quantities in place of the original numbers in a problem.<br>• Students monitor calculations, procedures and intermediate results while solving problems.<br>• Students verify and check if solutions are viable, or reasonable, within the context or situation. *(MTR.7.1)*<br>• Students reflect on the accuracy of their estimations and their solutions. | • Teacher provides opportunities for students to estimate or predict solutions prior to solving.<br>• Teacher encourages students to compare results to estimations and revise if necessary for future situations. *(MTR.5.1)*<br>• Teacher prompts students to self-monitor by continually asking, "Does this solution or intermediate result make sense? How do you know?"<br>• Teacher encourages students to provide explanations and justifications for results to self and others. *(MTR.4.1)* |

| **MTR** | **Student Moves** | **Teacher Moves** |
|---|---|---|
| MA.K12.MTR.7.1 *Apply mathematics to real-world contexts.* | • Students connect mathematical concepts to everyday experiences.<br>• Students use mathematical models and methods to understand, represent and solve real-world problems.<br>• Students investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them.<br>• Students re-design models and methods to improve accuracy or efficiency. | • Teacher provides real-world context to help students build understanding of abstract mathematical ideas.<br>• Teacher encourages students to assess the validity and accuracy of mathematical models and situations in real-world context, and to revise those models if necessary.<br>• Teacher provides opportunities for students to investigate, research and gather data to determine if a mathematical model is appropriate for a given situation from the world around them.<br>• Teacher provides opportunities for students to apply concepts to other content areas. |

# Algebra I-B Areas of Emphasis

In Algebra I-B, instructional time will emphasize five areas:

(1) performing operations with polynomials and radicals, and extending the Laws of Exponents to include rational exponents;
(2) extending understanding of functions to linear, quadratic and exponential functions and using them to model and analyze real-world relationships;
(3) solving quadratic equations in one variable; and
(4) building functions, identifying their key features and representing them in various ways

The purpose of the areas of emphasis is not to guide specific units of learning and instruction, but rather provide insight on major mathematical topics that will be covered within this mathematics course. In addition to its purpose, the areas of emphasis are built on the following:

- Supports the intentional horizontal progression within the strands and across the strands in this grade level or course.
- Student learning and instruction should not focus on the stated areas of emphasis as individual units.
- Areas of emphasis are addressed within standards and benchmarks throughout the course so that students are making connections throughout the school year.
- Some benchmarks can be organized within more than one area.
- Supports the communication of major mathematical topics to all stakeholders.
- Benchmarks within the emphasis areas should not be taught in the order they appear in. To do so would strip the progression of mathematical ideas and miss the opportunity to enhance horizontal progressions within the grade level or course.

The table below shows how the benchmarks within this mathematics course are embedded within the areas of emphasis.

| | | Operations with Polynomials and Radicals and Laws of Exponents | Linear, Quadratic and Exponential Functions | Solving Equations and Systems of Linear Equations and Inequalities | Building Functions, Identifying Key Features and Various Representations | Representing and Interpreting Categorical and Numerical Data |
|---|---|---|---|---|---|---|
| **Number Sense & Operations** | MA.912.NSO.1.1 | x | x | x | x | |
| | MA.912.NSO.1.2 | x | | | x | |
| | MA.912.NSO.1.4 | x | | x | | |
| **Algebraic Reasoning** | MA.912.AR.1.1 | x | x | x | x | x |
| | MA.912.AR.1.2 | | | x | | |
| | MA.912.AR.1.3 | x | | | | |
| | MA.912.AR.1.4 | x | | | | |
| | MA.912.AR.1.7 | x | x | x | x | |
| | MA.912.AR.3.1 | x | | x | | |
| | MA.912.AR.3.4 | | x | | | |
| | MA.912.AR.3.5 | | x | | | |
| | MA.912.AR.3.6 | | x | | x | |
| | MA.912.AR.3.7 | | x | | x | |
| | MA.912.AR.3.8 | | x | | x | |
| | MA.912.AR.5.3 | | x | | | |
| | MA.912.AR.5.4 | | x | | x | |
| | MA.912.AR.5.6 | | x | | x | |
| | MA.912.AR.9.6 | | | x | | |
| **Functions** | MA.912.F.1.1 | | x | | x | x |
| | MA.912.F.1.2 | x | | | x | |
| | MA.912.F.1.3 | | | | x | |
| | MA.912.F.1.6 | | x | | x | |
| | MA.912.F.1.8 | | x | | x | x |
| | MA.912.F.2.1 | | x | | x | |
| **Financial Literacy** | MA.912.FL.3.2 | | x | | | |
| | MA.912.FL.3.4 | | x | | x | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Data Analysis & Probability | MA.912.DP.1.1 | | | | | x |
| | | MA.912.DP.1.2 | | | | | x |
| | | MA.912.DP.1.4 | | | | | x |
| | | MA.912.DP.3.1 | | | | | x |

## Number Sense and Operations

**MA.912.NSO.1** *Generate equivalent expressions and perform operations with expressions involving exponents, radicals or logarithms.*

### *MA.912.NSO.1.1*

#### Benchmark

**MA.912.NSO.1.1** **Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.**

Benchmark Clarifications:
*Clarification 1:* Instruction includes the use of technology when appropriate.
*Clarification 2:* Refer to the K-12 Formulas (Appendix E) for the Laws of Exponents.
*Clarification 3:* Instruction includes converting between expressions involving rational exponents and expressions involving radicals.
*Clarification 4:* Within the Mathematics for Data and Financial Literacy course, it is not the expectation to generate equivalent numerical expressions

#### Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.5.3
- MA.912.FL.3.2

#### Terms from the K-12 Glossary

- Base
- Exponent
- Expression
- Rational Number

#### Vertical Alignment

**Previous Benchmarks**
- MA.6.NSO.3.3
- MA.7.NSO.1.1
- MA.8.NSO.1.3

**Next Benchmarks**
- MA.912.NSO.1.6

#### Purpose and Instructional Strategies

In grade 8, students generated equivalent numerical expressions and evaluated expressions using the Laws of Exponents with integer exponents. In Algebra I, students work with rational-number exponents. In later courses, students extend the Laws of Exponents to properties of logarithms.
- Instruction includes using the terms Laws of Exponents and properties of exponents interchangeably.
- Instruction includes student discovery of the patterns and the connection to mathematical operations and the inverse relationship between powers and radicals *(MTR.5.1)*.
- Problem types include having a fraction, integer or whole number as an exponent.
- Students should make the connection of the root being equivalent to a unit fraction exponent *(MTR.4.1)*

- For example, $\sqrt[3]{8} = \sqrt[3]{2^3}$ is equivalent to the equation $\sqrt[3]{8} = (2^3)^{\frac{1}{3}}$ which is equivalent to the equation which is equivalent to the equation which is equivalent to the equation $3\sqrt{8} = 2$.

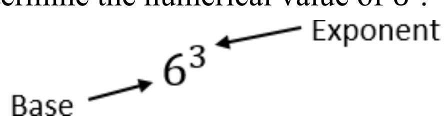
- When evaluating, students should be encouraged to approach from different entry points and discuss how they are different but equivalent strategies (MTR 2.1).
  - For example, if evaluating $(-27)^{\frac{2}{3}}$ students can:
    - Take the cube root of $-27$ first $\left(\sqrt[3]{-27}\right)^2$, or
    - Raise $-27$ to the second power first $\sqrt[3]{(-27)^2}$.

## Common Misconceptions or Errors

- Students may not understand the difference between an expression and an equation.
- Students may try to perform operations on bases as well as exponents.
- Students may multiply the base by the exponent instead of understanding that the exponent is the number of times the base occurs as a factor.
- Students may not truly understand exponents that are zero or negative.
- Students may not understand operations with rational numbers.

## Strategies to Support Tiered Instruction

- Teacher provides a review of the relationship between the base and the exponent by modeling an example of operations using a base and exponent.
  - For example, determine the numerical value of $6^3$.




$6^3$ which is equivalent to $6 \cdot 6 \cdot 6$ which is equivalent to 216.

- Teacher provides exploration of the rules of exponents through patterns. A strategy for developing meaning for integer exponents by making use of patterns is shown below:

**Patterns in Exponents**

| | |
|---|---|
| $5^5$ | $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ |
| $5^4$ | $5 \cdot 5 \cdot 5 \cdot 5$ |
| $5^3$ | $5 \cdot 5 \cdot 5$ |
| $5^2$ | $5 \cdot 5$ |
| $5^1$ | $5$ |
| $5^0$ | $1$ |
| $5^{-1}$ | $\frac{1}{5}$ |
| $5^{-2}$ | $\frac{1}{5 \cdot 5}$ |
| $5^{-3}$ | $\frac{1}{5 \cdot 5 \cdot 5}$ |
| $5^{-4}$ | $\frac{1}{5 \cdot 5 \cdot 5 \cdot 5}$ |

| | |
|---|---|
| $5^{-5}$ | $\frac{1}{5\cdot5\cdot5\cdot5\cdot5}$ |

- Teacher provides exploration of the rules of rational exponents through patterns. A strategy for developing meaning for rational exponents by making use of patterns is shown below:

**Patterns in Rational Exponents**

| | | |
|---|---|---|
| $5^{\frac{5}{2}}$ | $\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}$ | $\sqrt{5\cdot5\cdot5\cdot5\cdot5}$ |
| $5^{\frac{4}{2}}$ | $\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}$ | $\sqrt{5\cdot5\cdot5\cdot5}$ |
| $5^{\frac{3}{2}}$ | $\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}$ | $\sqrt{5\cdot5\cdot5}$ |
| $5^{\frac{2}{2}}$ | $\sqrt{5}\cdot\sqrt{5}$ | $\sqrt{5\cdot5}$ |
| $5^{\frac{1}{2}}$ | $\sqrt{5}$ | $\sqrt{5}$ |
| $5^{0}$ | $\sqrt{1}$ or 1 | $\sqrt{1}$ or 1 |
| $5^{-\frac{1}{2}}$ | $\frac{1}{\sqrt{5}}$ | $\sqrt{\frac{1}{5}}$ or $\frac{1}{\sqrt{5}}$ |
| $5^{-\frac{2}{2}}$ | $\frac{1}{\sqrt{5}\cdot\sqrt{5}}$ | $\sqrt{\frac{1}{5\cdot5}}$ or $\frac{1}{\sqrt{5\cdot5}}$ |
| $5^{-\frac{3}{2}}$ | $\frac{1}{\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}}$ | $\sqrt{\frac{1}{5\cdot5\cdot5}}$ or $\frac{1}{\sqrt{5\cdot5\cdot5}}$ |
| $5^{-\frac{4}{2}}$ | $\frac{1}{\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}}$ | $\sqrt{\frac{1}{5\cdot5\cdot5\cdot5}}$ or $\frac{1}{\sqrt{5\cdot5\cdot5\cdot5}}$ |
| $5^{-\frac{5}{2}}$ | $\frac{1}{\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}}$ | $\sqrt{\frac{1}{5\cdot5\cdot5\cdot5\cdot5}}$ or $\frac{1}{\sqrt{5\cdot5\cdot5\cdot5\cdot5}}$ |

- Instruction includes the use of fraction tiles to represent operations with positive fractions while simultaneously recording the equivalent numerical expressions.

## Instructional Tasks

*Instructional Task 1 (MTR.4.1, MTR.5.1)*

Part A. Think about when solving an equation with a radical. What is the inverse operation of a square root? Of a cube root?

Part B. Given the expression $\sqrt[3]{27}$, express 27 as a prime number with natural-number exponent.

Part C. How can we use the information from Part A and B to convert $\sqrt[3]{27}$ to exponential form?

*Instructional Task 2 (MTR3.1, MTR.4.1)*

Part A. Evaluate $64^{\frac{1}{3}}$ by first writing 64 as a power of 2 and using the properties of exponents.
Part B. Evaluate $64$ using a calculator.
Part C. Explain your process in both Part A and Part B. Define powers with fractional exponents in your own words.

*Instructional Task 3 (MTR.3.1, MTR.5.1)*

Part A. Given $f(x) = 32^x$, evaluate $f(0)$, $f(0.2)$, $f(0.4)$, $f(0.8)$ and $f(1)$ without the use of a calculator.
Part B. Graph the function $f$ in the domain $0 \leq x \leq 1$.
Part C. Between which two values in Part A would $f(0.5)$ be? Which one would it be closer to on the graph and why?

## Instructional Items

*Instructional Item 1*

Evaluate the numerical expression $(64)^{\frac{4}{3}}$.

*Instructional Item 2*

Rewrite $8^{0.5} \cdot 2^{\frac{2}{5}}$ as a single power of 2.

*Instructional Item 3*

Choose all of the expressions that are equivalent to $7^{\frac{5}{12}}$.

a. $\{(49\}^{\frac{1}{3}})\ \{(7\}^{-\frac{1}{4}})$
b. $\{(7\}^{\frac{2}{3}})\ \{(7\}^{-\frac{1}{4}})$
c. $7\{(7\}^{-\frac{1}{4}})$
d. $\sqrt[5]{7^{12}}$
e. $\sqrt[12]{7^5}$

*Instructional Item 4*

Evaluate the numerical expression $\left(-\frac{729}{64}\right)^{-\frac{2}{3}}$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.NSO.1.2*

## Benchmark

**MA.912.NSO.1.2 Generate equivalent algebraic expressions using the properties of exponents.**

*Example:* The expression $1.5^{3t+2}$ is equivalent to the expression $2.25(1.5)^{3t}$ which is equivalent to $2.25(3.375)^{t}$.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1, MA.912.AR.1.3,
- MA.912.AR.1.4, MA.912.AR.1.7
- MA.912.AR.5.3
- MA.912.FL.3.2

## Terms from the K-12 Glossary

- Base
- Expression
- Exponent

## Vertical Alignment

**Previous Benchmarks**

- MA.8.AR.1.1

**Next Benchmarks**

- MA.912.NSO.1.7

## Purpose and Instructional Strategies

In grade 8, students generated equivalent algebraic expressions using the Laws of Exponents with integer exponents. In Algebra I, students expand this work to include rational-number exponents. In later courses, students extend the Laws of Exponents to algebraic expressions with logarithms.

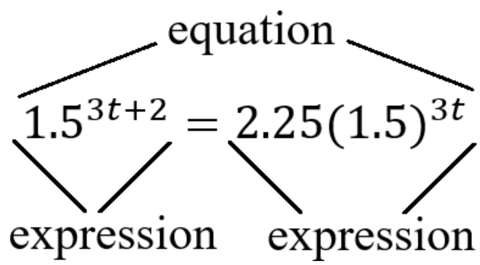
- Instruction includes using the terms Laws of Exponents and properties of exponents interchangeably.
- Instruction includes student discovery of the patterns and the connection to mathematical operations *(MTR.8.1)*.
- Students should be able to fluently apply the Laws of Exponents in both directions.
    - For example, students should recognize that $a^6$ is the quantity $(a^3)^2$; this may helpful when students are factoring a difference of squares.
- When generating equivalent expressions, students should be encouraged to approach from different entry points and discuss how they are different but equivalent strategies *(MTR.2.1)*.
- The expectation for this benchmark does not include the conversion of an algebraic expression from exponential form to radical form and from radical form to exponential form.

## Common Misconceptions or Errors

- Students may not understand the difference between an expression and an equation.
- Students may not have fully mastered the Laws of Exponents and understand the mathematical connections between the bases and the exponents.
- Student may believe that with the introduction of variables, the properties of exponents differ from numerical expressions.

## Strategies to Support Tiered Instruction

- Instruction includes the opportunity to distinguish between an expression and an equation. These should be captured in a math journal.
    - For example, when generating equivalent expressions, place an equal sign in between the expressions and label each expression and the equation.

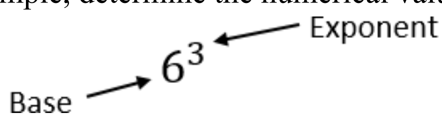



- Instruction provides opportunities to write each term in expanded form first and then use Laws of Exponents to combine like factors. It may also be helpful to chunk each step.
    - For example, to rewrite the expression $(8x^3)^2$ with one exponent, write out $(8)(x)(x)(x)(8)(x)(x)(x)$ and then use the commutative property to write $(8)(8)(x)(x)(x)(x)(x)(x) = 64x^6$.
- Teacher provides instruction for problems that require multiple applications of the Laws of Exponents by chunking the steps so that students are applying one property at each time and explaining the property applied. Each time ask students to identify the property of exponents that they applied.
- Teacher provides students side-by-side problems, one with variable bases and the other choosing a value for the variable. As students work through the problems, ask them about the similarities in the problem-solving process.
    - For example, teacher can model generating equivalent expressions like the ones below.

| | |
|---|---|
| $\left(\frac{5(3)^4}{4(3)^2}\right)^{0.5} =$ | $\left(\frac{5(x)^4}{4(x)^2}\right)^{0.5} =$ |
| $\frac{5^{0.5}3^{4\cdot 0.5}}{4^{0.5}(3)^{2\cdot 0.5}} =$ | $\frac{5^{0.5}x^{4\cdot 0.5}}{4^{0.5}(x)^{2\cdot 0.5}} =$ |
| $\frac{5^{0.5}3^2}{2(3)^1} =$ | $\frac{5^{0.5}x^2}{2(x)^1} =$ |

| | |
|---|---|
| $\frac{\{5^{0.5}(3\}^{2-1})}{2} =$ | $\frac{\{5^{0.5}(x\}^{2-1})}{2} =$ |
| $\frac{\{5^{0.5}(3\}^{1})}{2} =$ | $\frac{\{5^{0.5}(x\}^{1})}{2} =$ |
| $\frac{3\sqrt{5}}{2}$ | $\frac{x\sqrt{5}}{2}$ |

- Teacher provides a review of the relationship between the base and the exponent by modeling an example of operations using a base and exponent.
    - For example, determine the numerical value of $6^3$.




$6^3$ which is equivalent to $6 \cdot 6 \cdot 6$ which is equivalent to 216.

## Instructional Tasks

*Instructional Task 2 (MTR.3.1, MTR.4.1)*

Part A. Write the algebraic expression $\left(\frac{6x^{\frac{2}{7}}y\, z^0}{9x^2y^5\, z^{-8}}\right)^3$ as an equivalent expression where each variable only appears once.

Part B. Compare your method of simplifying with a partner.

*Instructional Task 2 (MTR.3.1, MTR.4.1)*

Part A. Describe the steps taken to simplify the following expression: $\frac{5a^4b^3}{(2ab)^2}$ using exponent rules.

Part B. Which rules were used in each step?

*Instructional Task 3(MTR.3.1, MTR.4.1, MTR.6.1)*

Part A. Consider the following expression: $\frac{4x^0\, y^{-2}\, z^3}{4x}$ and explain the first rule that would be used to simplify the expression.

Part B. Why did you choose that rule? Compare your answer with a partner.

## Instructional Items

*Instructional Item 1*

Given the algebraic expression $2.3^{2t-1}$, create an equivalent expression.

*Instructional Item 2*

Use the properties of exponents to create an equivalent expression for the given expression shown below with no variables in the denominator.

$$(64x^2)\text{-}(32x^5)$$

*Instructional Item 3*

Create an expression equivalent to the expression $\left(\frac{12x^{-5}}{7z^{-3}}\right)^{-10}$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.NSO.1.4*

## Benchmark

**MA.912.NSO.1.4 Apply previous understanding of operations with rational numbers to add, subtract, multiply and divide numerical radicals.**

*Algebra I Example:* The expression $\frac{\sqrt{136}}{\sqrt{2}}$ is equivalent to $\sqrt{\frac{136}{2}}$ which is equivalent to $\sqrt{68}$ which is equivalent to $2\sqrt{17}$.

Benchmark Clarifications:

*Clarification 1:* Within the Algebra I course, expressions are limited to a single arithmetic operation involving two square roots or two cube roots.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1, MA.912.AR.1.2

## Terms from the K-12 Glossary

- Base
- Rational Number

## Vertical Alignment

**Previous Benchmarks**

- MA.8.NSO.1.5

**Next Benchmarks**

- MA.912.NSO.1.5
- MA.912.AR.7.1, MA.912.AR.7.4

## Purpose and Instructional Strategies

In grade 8, students evaluated numerical expressions with square and cube roots. In Algebra I, students perform operations with numerical square or cube roots. In later courses, students will perform operations with algebraic expressions involving radicals.

- It is important to reinforce and activate the prior knowledge of simple calculations with radicals within this benchmark.
- Within this benchmark, it is not the expectation to rationalize the denominator.

However, students should have experience and understanding of how to rationalize the denominator. Students should understand when it may be helpful to rationalize, for example when estimating values.

- Instruction includes making the connection to properties of exponents with rational exponents.
  - For example, when determining the value of the expression $4\sqrt{3}(\sqrt{3})$, a student could convert to exponential notation, which would result in $4\left(3^{\frac{1}{2}}\right) \text{ x } 3^{\frac{1}{2}}$. A student could then use properties of exponents to obtain $4\left(3^{\frac{1}{2}+\frac{1}{2}}\right)$ which is equivalent to 12.
  - For example, when determining the value of the expression $\frac{4\sqrt{3}}{\sqrt{3}}$, a student could convert to exponential notation, which would result in $\frac{4\left(3^{\frac{1}{2}}\right)}{3^{\frac{1}{2}}}$. A student could then use properties of exponents to obtain $4\left(3^{\frac{1}{2}-\frac{1}{2}}\right)$ which is equivalent to 4.

- Instruction allows students to write their answer in radical or exponential form.
- Instruction may include use of the words
  - Radicand – the value or expression written under the radical sign
  - Index – the value that indicates the root of the radicand to be taken

$$\sqrt[\text{index}]{radicand}$$

## Common Misconceptions or Errors

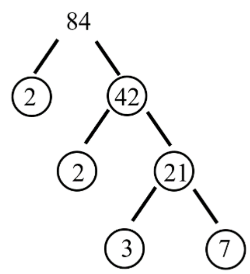
- Students may not know how to do simple calculations with radicals; therefore, they may not take the square root of the perfect square factor; or they may suggest using a factor pair within a radical that does not contain a perfect square as a factor.
- Students may confuse radicands and coefficients and perform the operations on the wrong part of the expression.

## Strategies to Support Tiered Instruction

- Teacher provides opportunities to write out all of the factors, as sets, of the radicand. Once the factors have been identified, the students can use the set of factors with the highest perfect square or cube, as applicable.
  - For example, given $\sqrt{200}$, the radicand is 200, the factor sets are {1 and 200}, {2 and 100}, {4 and 50}, etc., 100 is the highest perfect square; so, $\sqrt{200} = 10\sqrt{20}$

- Teacher models prime factorization as a strategy to find all factors of radicand.

$\sqrt{84}$

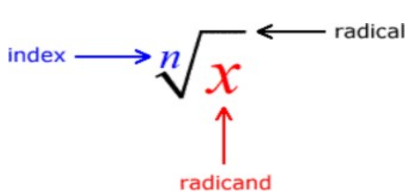


$\sqrt{2^2 \cdot 3 \cdot 7} = 2\sqrt{21}$

- Teacher co-creates a list of common perfect squares and cubes that can be used as problems are performed.
  - For example, a list of common perfect squares and cubes is shown.

| | **Perfect Squares** | **Perfect Cubes** |
|---|---|---|
| 1 | $1^2 = 1$ | $1^3 = 1$ |
| 2 | $2^2 = 4$ | $2^3 = 8$ |
| 3 | $3^2 = 9$ | $3^3 = 27$ |
| 4 | $4^2 = 16$ | $4^3 = 64$ |
| 5 | $5^2 = 25$ | $5^3 = 125$ |
| 6 | $6^2 = 36$ | $6^3 = 216$ |
| 7 | $7^2 = 49$ | $7^3 = 343$ |

- Teacher models identifying all parts of the radical expression and rules specific to the operation prior to calculating answers. To add or subtract radicals the radicand must be the same prior to adding the coefficients.
  - For example, given $6\sqrt{20} + 4\sqrt{20}$, 6 and 4 are coefficients and 20 is the radicand for both terms. Therefore, $6\sqrt{20} + 4\sqrt{20} = 10\sqrt{20}$.
  - For example, given $4\sqrt{5} + 9\sqrt{20}$, 4 and 9 are coefficients and 5 and 20 are the radicands. Therefore, the terms cannot be combined as is. But notice that if $\sqrt{20}$ is rewritten as $2\sqrt{5}$, then 5 is the radicand for both terms. So, $4\sqrt{5} - 9\sqrt{20}$, which is equivalent to $4\sqrt{5} - 18\sqrt{5}$ which when subtracted results in $-14\sqrt{5}$.
- Teacher provides a visual aid (e.g., laminated cue card) to distinguish the parts of a radical expression.
  - For example, in the expression $\sqrt[3]{20}$, 3 is the index (or root) and 20 is the radicand.



## Instructional Tasks

*Instructional Task 1 (MTR.3.1, MTR.7.1)*

The velocity, $v$, measured in meters per second of an object can be measured in terms of its mass, $m$, measured in kilograms and Kinetic Energy, $E_k$, measured in Joules. The equation below describes this relationship.

$$v = \sqrt{\frac{2\mathrm{E_k}}{\mathrm{m}}}$$

Part A. What is the exact velocity of an object if its mass measures 31 kilograms and its Kinetic Energy measures 310 Joules?
Part B. Rearrange the formula to highlight mass as the quantity of interest.
Part C. What is the mass of an object if the velocity measures 6.1 meters per second and its Kinetic Energy measures 31.4 Joules?

## Instructional Items

*Instructional Item 1*

Determine the value of the expression $\sqrt[3]{24} + \sqrt[3]{81}$ Write your answer as an exact quantity using only a single radical.

*Instructional Item 2*

Determine the difference of the expression $\sqrt{24} - 2\sqrt{54}$. Write your answer as an exact quantity using only a single radical.

*Instructional Item 3*

Determine the value of the expression $4\sqrt{10}\left(-3\sqrt{15}\right)$. Write your answer as an exact quantity using only a single radical.

*Instructional Item 4*

Determine the value of the expression $\frac{3\sqrt{3}}{5\sqrt{75}}$.

---

**The strategies, tasks and items included in the B1G-M are examples and should not be considered*

## Algebraic Reasoning

**MA.912.AR.1** *Interpret and rewrite algebraic expressions and equations in equivalent forms.*

### *MA.912.AR.1.1*

#### Benchmark

**MA.912.AR.1.1** **Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity.**

*Algebra I Example:* Derrick is using the formula $P = 1000(1 + .1)^t$ to make a prediction about the camel population in Australia. He identifies the growth factor as $(1 + .1)$, or 1.1, and states that the camel population will grow at an annual rate of 10% per year.

*Example:* The expression $1.15t$ can be rewritten as $\{(1.15\}^{\frac{1}{12}})^{12t}$ which is approximately equivalent to $1.012^{12t}$. This latter expression reveals the approximate equivalent monthly interest rate of 1.2% if the annual rate is 15%.

<u>Benchmark Clarifications:</u>
*Clarification 1*: Parts of an expression include factors, terms, constants, coefficients and variables.
*Clarification 2:* Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

#### Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.2
- MA.912.AR.2.2, MA.912.AR.2.5, MA.912.AR.2.6
- MA.912.AR.3.1, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.1
- MA.912.AR.5.3, MA.912.AR.5.6
- MA.912.FL.3.2

#### Terms from the K-12 Glossary

- Coefficient
- Expression
- Equation

#### Vertical Alignment

**Previous Benchmarks**

- MA.8.AR.2.1, MA.8.AR.2.2

**Next Benchmarks**

- MA.912.AR.5.5, MA.912.AR.5.9
- MA.912.AR.8.2
- MA.912.T.3.2

## Purpose and Instructional Strategies

In grade 8, students generated and identified equivalent linear expressions, and solved multi-step problems involving linear expressions within real-world contexts. In Algebra I, students generate and interpret equivalent linear, absolute value, quadratic and exponential expressions and equations. In later courses, students will identify and interpret other functional (exponential, rational, logarithmic, trigonometric, etc.) expressions and equations.

- Instruction includes making the connection to linear, absolute value, quadratic and exponential functions.
    - Students should be able to identify factors, terms, constants, coefficients and variables in expressions and equations.
        - Go beyond these popular parts of an expression and equation: the growth/decay factor in exponential functions, rate of change in linear functions, interest, etc.
    - Look for opportunities to interpret these components in context – make these discussions part of daily instruction.

## Common Misconceptions or Errors

- Students may not be able to identify parts of an expression and equation or interpret those parts within context. Ensure these are embedded throughout instruction and discussions.
    - For example, building in questions to identify these parts and discussing their connection to the context in which they represent in a routine way will help students to make these connections.

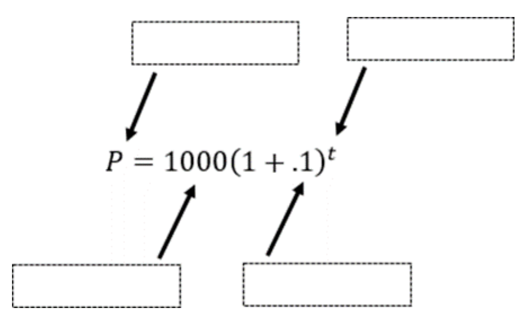
## Strategies to Support Tiered Instruction

- Teacher facilitates discussions which include questions and clarifications to identify the connections of expressions and equations to the context of problems.
- Instruction provides opportunities to increase understanding of vocabulary terms.
    - For example, instruction may include a vocabulary review using a chart shown.

| | |
|---|---|
| Term | $6x$ |
| Coefficient | $6x$ |
| Variable | $x$ |
| Constant | $6$ |

- Teacher provides students with an expression or equation and allows them to match the parts to key vocabulary.
    - For example, teacher can provide the word bank to identify the different parts of the equation shown.

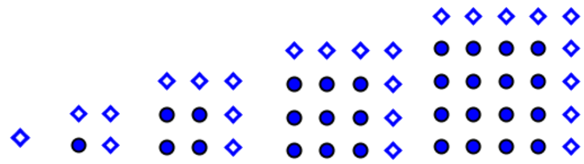
| **Word Bank** |
|---|
| Initial amount/value |
| Final amount/value |
| Rate of growth |
| Rate of decay |
| Time |



## Instructional Tasks

*Instructional Task 1 (MTR.5.1)*

The algebraic expression $(n - 1)^2 + (2n - 1)$ can be used to calculate the number of symbols in each diagram below. Explain what $n$ likely represents, how the parts of this expression relate to the diagrams, and why the expression results in the number of symbols in each diagram.



*Instructional Task 2 (MTR.3.1, MTR.7.1)*

Last weekend, Cindy purchased two tops, a pair of pants, and a skirt at her favorite store. The equation $T = 1.075x$ can be used to calculate her total cost where $x$ represents the pretax subtotal cost of her purchase.

Part A. In the equation $T = 1.075x$, what does the number 1 represent? Explain using the context of Cindy's situation.

Part B. In the equation $T = 1.075x$, what does the number 0.075 represent? Explain using the context of Cindy's situation.

## Instructional Items

*Instructional Item 1*

Identify the factors in the expression $2(3x - 1) + 2(2x + 2)$.

*Instructional Item 2*

A bacteria's growth can be modeled by the equation $y=10(1 + 0.3)^{(t)}$ Identify the value that represents the starting amount of bacteria.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.AR.1.2*

## Benchmark

**MA.912.AR.1.2 Rearrange equations or formulas to isolate a quantity of interest.**

*Algebra I Example:* The Ideal Gas Law $PV = nRT$ can be rearranged as $T = \frac{PV}{nR}$ to isolate temperature as the quantity of interest.

*Example:* Given the Compound Interest formula $A = P(1 + \frac{r}{n})^{nt}$, solve for $P$.

*Mathematics for Data and Financial Literacy Honors* Example: Given the Compound Interest formula $A = P(1 + \frac{r}{n})^{nt}$, solve for $t$.

Benchmark Clarifications:
*Clarification 1*: Instruction includes using formulas for temperature, perimeter, area and volume; using equations for linear (standard, slope-intercept and point-slope forms) and quadratic (standard, factored and vertex forms) functions.
*Clarification 2:* Within the Mathematics for Data and Financial Literacy course, problem types focus on money and business.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.2
- MA.912.AR.2.2, MA.912.AR.2.5, MA.912.AR.2.6
- MA.912.AR.3.1, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.1
- MA.912.AR.5.3, MA.912.AR.5.6
- MA.912.FL.3.2

## Terms from the K-12 Glossary

- Equation

## Vertical Alignment

**Previous Benchmarks**
- MA.8.AR.2, MA.8.AR.2.3
- MA.8.GR.1

**Next Benchmarks**
- MA.912.AR.5.5, MA.912.AR.5.9
- MA.912.AR.8.2
- MA.912.T.3.2

## Purpose and Instructional Strategies

In grade 8, students isolated variables in one-variable linear equations and one-variable quadratic equations in the form $x^2 = p$ and $x^3 = q$. In Algebra I, students isolate a variable or quantity of interest in equations and formulas. Equations and variables will focus on linear, absolute value and quadratic in Algebra I. In later courses, students will highlight a variable or quantity of interest for other types of equations and formulas, including exponential, logarithmic and trigonometric.

- Instruction includes making connections to inverse arithmetic operations (refer to Appendix D) and solving one-variable equations.

| | |
|---|---|
| $4x - 2y = 8$ | Given |
| $-4x + 4x - 2y = 8 - 4x$ | Subtraction Property of Equality |
| $-2y = 8 - 4x$ | Simplify |

$$\frac{-2y}{-2} = \frac{8}{-2} - \frac{4x}{-2}$$ Division Property of Equality

$$y = -4 + 2x$$ Simplify

- Instruction includes justifying each step while rearranging an equation or formula.
  - For example, when rearranging $A = P\,(1 + \frac{r}{n})$ for $P$, it may be helpful for students to highlight the quantity of interest with a highlighter, so students remain focused on that quantity for isolation purposes. It may also be helpful for students to identify factors, or other parts of the equations.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

P and (1) are factors so the inverse operation is division.

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

## Common Misconceptions or Errors

- Students may not have mastered the inverse arithmetic operations.
- Students may be frustrated because they are not arriving at a numerical value as their solution. Remind students that they are rearranging variables that can be later evaluated to a numerical value.
- Having multiple variables and no values may confuse students and make it difficult for them to see the connections between rearranging a formula and solving a one-variable equation.

## Strategies to Support Tiered Instruction

- Instruction includes doing a side-by-side comparison of solving a multistep equation with rearranging equations and formulas. The teacher should allow students time to understand that the steps in solving both equations are the same.
  - For example, solve both equations and note the similarities in solving both types of equations.

| Solving One-Variable Equations | Rearranging Equations/Formulas |
|---|---|
| Determine the height, $h$, in the formula $SA = 2B + Ph$ if the surface area ($SA$) is 537 units squared, the area of the base ($B$) is 112 units squared and the perimeter of the base ($P$) is 25 units.<br><br>$537 = 2(112) + 25h$<br>$537 - 22 = 224 + 25h - 224$<br>$313 = 25h$<br>$\frac{313}{25} = \frac{25h}{25}$<br>$12.52 = h$ | Isolate height, $h$, in the formula $SA = 2B + Ph$.<br><br>$SA = 2B + Ph$<br>$SA - 2B = 2B + Ph - 2B$<br>$SA - 2B = Ph$<br>$\frac{SA - 2B}{P} = \frac{Ph}{P}$<br>$\frac{SA - 2B}{P} = h$ |

- Teacher provides a chart for students to use as a study guide or to copy in their interactive notebook.
  - For example, inverse operations chart below.

| Inverse Operations Chart | | |
|---|---|---|
| Addition<br>$+$ | $\leftrightarrow$ | Subtraction<br>$-$ |
| Multiplication<br>$\times$ | $\leftrightarrow$ | Division<br>$\div$ |
| Square<br>$x^2$ | $\leftrightarrow$ | Square Root<br>$\sqrt{}$ |

## Instructional Tasks

*Instructional Task 1 (MTR.4.1, MTR.5.1)*

Part A. Given the equation $ax^2 + bx + c = 0$, solve for $x$.
Part B. Share your strategy with a partner. What do you notice about the new equation(s)?

*Instructional Task 2 (MTR.4.1, MTR.5.1)*

Part A. Given the equation $Ax + By = C$, solve for B.
Part B. Given the equation $7x - 6y = 24$, determine the $x$- and $y$-intercepts.
Part C. What do you notice between Part A and Part B?

## Instructional Items

*Instructional Item 1*

Solve for $x$ in the equation $3x + y = 5x - xy$.

*Instructional Item 2*

The formula $d = \frac{v_o + v_t}{2}t$ relating to the translational of motion, where $d$ represents distance, $v_o$ represents initial velocity, $v_t$ represents final velocity, and $t$ represents time. Rearrange the formula to isolate final velocity.

*Instructional Item 3*

The area *A* of a sector of a circle with radius *r* and angle-measure *S* (in degrees) is given by $A = \frac{\pi r^2 S}{360}$ solve for the radius *r*.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

### *MA.912.AR.1.3*

## Benchmark

**MA.912.AR.1.3 Add, subtract and multiply polynomial expressions with rational number coefficients.**

Benchmark Clarifications:
*Clarification 1:* Instruction includes an understanding that when any of these operations are performed with polynomials the result is also a polynomial.
*Clarification 2:* Within the Algebra I course, polynomial expressions are limited to 3 or fewer terms.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1, MA.912.NSO.1.2
- MA.912.F.3.1

## Terms from the K-12 Glossary

- Polynomial
- Monomial

## Vertical Alignment

**Previous Benchmarks**
- MA.7.AR.1.1
- MA.8.AR.1.2
- MA.8.AR.1.3

**Next Benchmarks**
- MA.912.AR.1.5, MA.912.AR.1.6, MA.912.AR.1.7
- MA.912.AR.6.3

## Purpose and Instructional Strategies

In middles grades, students added, subtracted and multiplied linear expressions. In Algebra I, students perform operations on polynomials limited to 3 or fewer terms. In later courses, students will perform operations on all polynomials.

- Instruction includes making the connection to dividing a polynomial by a monomial and the understanding that division does not have closure (the result may or may not be another polynomial).
- Reinforce like terms during instruction (using different colors can be a strategy to help identify them as unique from one another).
- Instruction includes the use of manipulatives, like algebra tiles, and various strategies, like the area model, properties of exponents and the distributive property.
    - Area model

      The expression $(2x^2 + 1.5x + 6)(3x + 4.2)$ is equivalent to $6x^3 + 12.9x^2 + 24.3x + 25.2$ and can be modeled below.

| | $2x^2$ | $1.5x$ | $6$ |
|---|---|---|---|
| $3x$ | $6x^3$ | $4.5x^2$ | $18x$ |
| $4.2$ | $8.4x^2$ | $6.3x$ | $25.2$ |

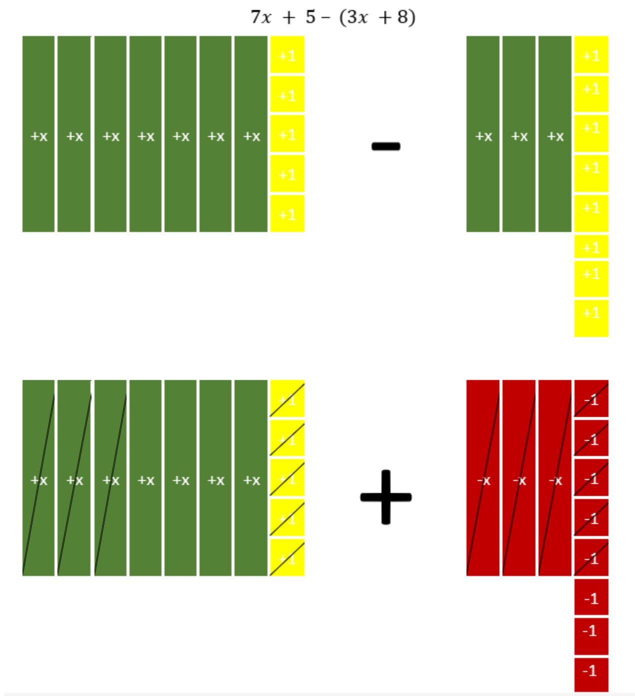
- Instruction should not rely upon the use of tricks or acronyms, like FOIL.
- Although within the Algebra I course, polynomial expressions are limited to 3 or fewer terms, this restriction only refers to the expressions given to the student, not the expression after the operation applied.

## Common Misconceptions or Errors

- Students may not understand the meaning of closure or the operations it applies to with polynomials.
- Students may not understand like terms or the properties of exponents.
- Students may not distribute the factor of -1 when subtracting a polynomial.
- Student might multiply incorrectly when squaring a binomial. For example, $(x - h)^2$ incorrectly distributed as $x^2 - h^2x^2 - h^2$ instead of $x^2 - 2\text{xh} + h^2x^2 - 2\text{xh} + h^2$

## Strategies to Support Tiered Instruction

- Teacher models subtracting polynomials with algebra tiles.




- Instruction includes converting a quadratic from vertex form to standard form. Instruct students to write out $(x - h)^2$ as $(x - h)(x - h)$. Remind students that different methods can be used to multiply binomials.
- Instruction includes the use of shapes or colors to demonstrate like terms. Teacher must ensure that students understand that the sign in front of the terms are key when combining like terms.
    - For example, different colors can be used when adding the polynomials shown below.

$$(x^2 - 4x + 12) + (3x^2 + 7x - 4)$$
$$(x^2 - 4x + 12) + (3x^2 + 7x - 4)$$
$$x^2 + 3x^2 - 4x + 7x + 12 - 4$$
$$4x^2 + 3x + 8$$

- Teacher provides examples showing that polynomials are closed under the operations of addition, subtraction and multiplication, but not under division.
    - For example, when subtracting or multiplying polynomials (as shown below), the result is always a polynomial.

$$7x + 5 - (3x + 8) = 4x - 3$$
$$(7x + 5)(3x + 8) = 21x^2 + 71x + 40$$

- For example, when dividing polynomials (as shown below), the result may or may not be a polynomial.

$$\frac{2x^2+5x}{x} = 2x + 5 \text{ (which is a polynomial)}$$

$$\frac{2x^2+5x+1}{x} = 2x + 5 + \frac{1}{x} \text{ (which is not a polynomial)}$$

## Instructional Tasks

*Instructional Task 1 (MTR.3.1, MTR.4.1)*

Part A. Determine the sum of $3x^2 - 2x + 5$ and $\frac{1}{6}x^2 + 7x + \frac{8}{7}$. Explain the method used in determining the sum.

Part B. Discuss whether the addition of polynomials will always result in another polynomial. Why or why not?

Part C. Determine the difference of $3x^3 - 2x^2 + 5$ and $x^2 - 0.25x + 1.24$. Explain the method used in determining the difference.

Part D. Discuss whether the subtraction of polynomials will always result in another polynomial. Why or why not?

Part E. Determine the product of $2x + 5$ and $\frac{2}{9}x^2 - \frac{11}{2}x + 1$. Explain the method used in determining the product.

Part F. Discuss whether the multiplication of polynomials will always result in another polynomial. Why or why not?

Part G. Determine the quotient of $9x^2 - 3x + 12$ and $3x$. Explain the method used in determining the quotient.

Part H. Discuss whether the division of polynomials will always result in another polynomial. Why or why not?

## Instructional Items

*Instructional Item 1*

Determine the sum of the expression $(\frac{3}{4}x^3 - \frac{2}{3}) + (-\frac{1}{2}x^2 + x + \frac{5}{6})$.

*Instructional Item 2*

Determine the coefficient of the $x^2$ term when the expression $(x^2 + \frac{3}{4}x - \frac{1}{2})$ is multiplied by $\left(x - \frac{2}{3}\right)$.

*Instructional Item 3*

Determine the difference of the expression $(-0.4x + 0.5x^2 + 2) - (0.6 + x^2 + 0.5x)$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.AR.1.4*

## Benchmark

**MA.912.AR.1.4 Divide a polynomial expression by a monomial expression with rational number coefficients.**

Benchmark Clarifications:
*Clarification 1:* Within the Algebra I course, polynomial expressions are limited to 3 or fewer terms.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1
- MA.912.NSO.1.2
- MA.912.F.3.1

## Terms from the K-12 Glossary

- Monomial
- Polynomial

## Vertical Alignment

**Previous Benchmarks**

- MA.7.AR.1.1
- MA.8.AR.1.2
- MA.8.AR.1.3

**Next Benchmarks**

- MA.912.AR.1.5, MA.912.AR.1.6, MA.912.AR.1.7
- MA.912.AR.6.3

## Purpose and Instructional Strategies

In middles grades, students added, subtracted and multiplied linear expressions. In Algebra I, students perform operations on polynomials limited to 3 or fewer terms and divide polynomials by a monomial. In later courses, students will perform operations on all polynomials.

- Instruction includes the connection to addition, subtraction and multiplication of polynomials to develop the understanding of closure, and the connection to properties of exponents.
- Instruction includes proper vocabulary and terminology, keeping in mind that when dividing, the word "cancel" can become a misconception for students. A number or an expression divided by itself is equivalent to 1 and does not disappear (see Appendix D).
- Instruction includes the use of manipulatives, like algebra tiles, and various strategies, like the area model, properties of exponents and decomposing fractions.
    - Decomposing fractions

      The expression $\frac{12mn^6 - 40m^2n^3}{4m2n3}$ can be written as $\frac{12mn^6}{4m2n3} - \frac{40m^2n^3}{4m2n3}$. Students can then perform the division with each fraction to determine the difference.

## Common Misconceptions or Errors

- Students may not understand the meaning of closure (although not directly discussed in this benchmark, polynomials are not closed under division).
- Students may not understand like terms.

## Strategies to Support Tiered Instruction

- Teacher provides examples showing that polynomials are closed under the operations of addition, subtraction and multiplication, but not under division.
  - For example, when subtracting or multiplying polynomials (as shown below), the result is always a polynomial.

$$7x + 5 - (3x + 8) = 4x - 3$$
$$(7x + 5)(3x + 8) = 21x^2 + 71x + 40$$

  - For example, when dividing polynomials (as shown below), the result may or may not be a polynomial.

$\frac{2x^2+5x}{x} = 2x + 5$ (which is a polynomial)

$\frac{2x^2+5x+1}{x} = 2x + 5 + 1$ (which is not a polynomial)

## Instructional Tasks

*Instructional Task 1 (MTR.3.1, MTR.4.1, MTR.5.1)*

Part A. Determine the quotient of $(\frac{1}{3}a^4 - 3a^3 + \frac{1}{2}a^2)$ and $3a^3$.

Part B. Discuss with your partner the strategy used. How do your quotients compare to one another?

*Instructional Task 2 (MTR.3.1, MTR.4.1, MTR.5.1)*

Part A. Determine the quotient of $x + x^2$ and $x^{-1}$.

Part B. What do you notice about your answer and the Laws of Exponents?

Part C. What happens when you divide $x + x^2$ by the expression $x^{\frac{1}{2}}$ (which is not a monomial)?

## Instructional Items

*Instructional Item 1*

What is the quotient of the expression $\frac{15d^2-25d^4}{5d^3}$ ?

*Instructional Item 2*

What is the quotient of the expression $\frac{8m^2n-5mn^2-20mn}{\frac{1}{4}mn}$ ?

*Instructional Item 3*

What is the quotient of the expression $(-\frac{5}{12}x^3y + \frac{2}{3}x^2y^2 - \frac{1}{2}x^2y) \div (-6x^2y)$?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*



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*MA.912.AR.1.7*

## Benchmark

**MA.912.AR.1.7 Rewrite a polynomial expression as a product of polynomials over the real number system.**

*Example:* The expression $4x^3y - 3x^2y^4$ is equivalent to the factored form $x^2y(4x - 3y^3)$.
*Example:* The expression $16x^2 - 9y^2$ is equivalent to the factored form $(4x - 3y)(4x + 3y)$.

Benchmark Clarifications:
*Clarification 1:* Within the Algebra I course, polynomial expressions are limited to 4 or fewer terms with integer coefficients.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1
- MA.912.NSO.1.2
- MA.912.AR.3.1, MA.912.AR.3.5, MA.912.AR.3.6, MA.912.AR.3.7, MA.912.AR.3.8

## Terms from the K-12 Glossary

- Expression
- Polynomial
- Greatest Common Factor

## Vertical Alignment

**Previous Benchmarks**
- MA.6.AR.1.4
- MA.7.AR.1.2
- MA.8.AR.1.3

**Next Benchmarks**
- MA.912.AR.1.8

## Purpose and Instructional Strategies

In grade 8, students rewrote binomial algebraic expressions as a common factor times a binomial. In Algebra I, students rewrite polynomials, up to 4 terms, as a product of polynomials over the real numbers. In later grades, students will rewrite polynomials as a product of polynomials over real and complex number systems.

- Instruction includes special cases such as difference of squares and perfect square trinomials.
- Instruction builds upon student prior knowledge of factors, including greatest common factors.
- Instruction includes the student understanding that factoring is the inverse of multiplying polynomial expressions.
- Instruction includes the use of models, manipulatives and recognizing patterns when factoring.
    - Sum-Product Pattern
      The expression $x^2 + 7x + 10$ can be written as $(x + 5)(x + 2)$ since $5 + 2 = 7$ and $5(2) = 10$.

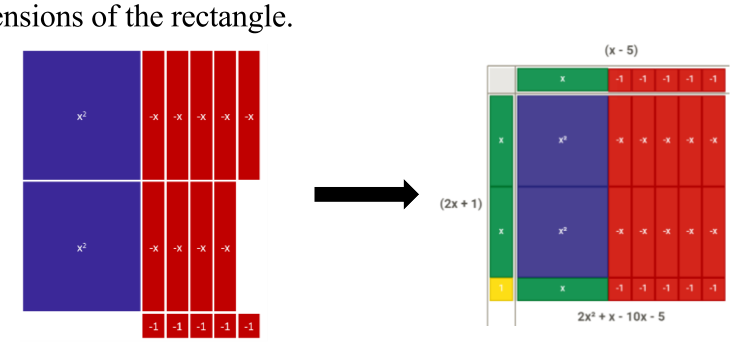
- Factor by Grouping
  The expression $x^3 + 7x^2 + 2x + 14$ can be grouped into two binomials and rewritten as $(x^3 + 7x^2) + (2x + 14)$. Each binomial can be factored and rewritten as $x^2(x + 7) + 2(x + 7)$ resulting in the same factor and the factored form as $(x^2 + 2)(x + 7)$.
- A-C Method
  When factoring trinomials $ax^2 + bx + c$, multiply $a$ and $c$, then determine factor pairs of the product. Using the factor pair that adds to $b$ and multiplies to $c$, rewrite the middle term and then factor by grouping.
  - For example, given $2x^2 + x - 6$ and that $ac = -12$, one can determine that two numbers that add to 1 and multiply to -12 are 4 and -3. This information can be used to rewrite the given quadratic as $2x^2 + 4x - 3x - 6$. Then, using factor by grouping the expression is equivalent to $(2x^2 + 4x) - (3x + 6)$ which is equivalent to $2x(x + 2) - 3(x + 2)$ which is equivalent to the factored form $(2x - 3)(x + 2)$.
- Area Model/Box Method
  To factor $ax^2 + bx + c$, the general box method is shown below.

| | | |
|---|---|---|
| | $ax^2$ | $b_1x$ |
| | $b_{2x}$ | $c$ |

  - For example, to factor $2x^2 - 9x - 5$ the box method is shown below. The first term, 2x^2, is placed in the top left box and the constant, -5, is placed in the bottom right box. Then reference the A-C Method or guess and check to determine how to separate the middle term, -9x into the remaining boxes. The factors can then be identified using the Greatest Common Factor across each row and column.

| | $2x$ | $1$ |
|---|---|---|
| $x$ | $2x^2$ | $1x$ |
| $-5$ | $-10x$ | $-5$ |

- Area Model/Box Method (Algebra tiles)
  The factorization of $2x^2 - 9x - 5$ using algebra tiles is shown below. Begin with the representation of the given polynomial and arrange the tiles into a complete rectangle, adding zero pairs of tiles if needed. The factors are then determined by the dimensions of the rectangle.



## Common Misconceptions or Errors

- Students may not identify the greatest common factor or factor completely.

## Strategies to Support Tiered Instruction

- Instruction includes providing a flow chart to reference while completing examples.
- Instruction includes providing definition of greatest common factor and strategies for identifying the greatest common factor of numerical or algebraic terms.
  - For example, the expression $8x^3 - 4x^2$ has common factors of 2 and $x$, but these are not greatest common factors. The greatest common factor of the coefficients is 4 and the greatest common factor of the variable terms is $x^2$. So, the greatest common factor of the two terms is $4x^2$. The expression $8x^3 - 4x^2$ can be rewritten as $4x^2(2x - 1)$.

## Instructional Tasks

*Instructional Task 1 (MTR.3.1, MTR.4.1, MTR.5.1)*

Part A. Given the polynomial $x^4 - 16y^4z^8$, rewrite it as a product of polynomials.
Part B. Discuss with your partner the strategy used. How do your polynomial factors compare to one another?

*Instructional Task 2 (MTR.3.1, MTR.5.1)*

Part A. What are the factors of the quadratic $16x^2 - 48x + 36$?
Part B. Determine the roots of the quadratic function $f(x) = 16x^2 - 48x + 36$.
Part C. What do you notice about your answers from Part A and Part B?
Part D. Graph the function $f(x) = 16x^2 - 48x + 36$.

## Instructional Items

*Instructional Item 1*

Given the polynomial $x^4 - 16y^4z^8$, rewrite it as a product of polynomials.

*Instructional Item 2*

Given the polynomial $x^2 - 10x + 24$, rewrite it as a product of polynomials.

*Instructional Item 3*

Given the polynomial $x^3 - 3x^2 - 9x + 27$, rewrite it as a product of polynomials.

*Instructional Item 4*

What is one of the factors of the polynomial $21r^3s^2 - 14r^2s + 14rs^3$?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

**MA.912.AR.3** *Write, solve and graph quadratic equations, functions and inequalities in one and two variables.*

*MA.912.AR.3.1*

### Benchmark

**MA.912.AR.3.1 Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.**

Benchmark Clarifications:
*Clarification 1:* Within the Algebra I course, instruction includes the concept of non-real answers, without determining non-real solutions.
*Clarification 2:* Within this benchmark, the expectation is to solve by factoring techniques, taking square roots, the quadratic formula and completing the square.

### Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.2, MA.912.AR.1.3, MA.912.AR.1.7

### Terms from the K-12 Glossary

- Coefficient
- Quadratic Equation
- Real Numbers
- $x$-intercept

### Vertical Alignment

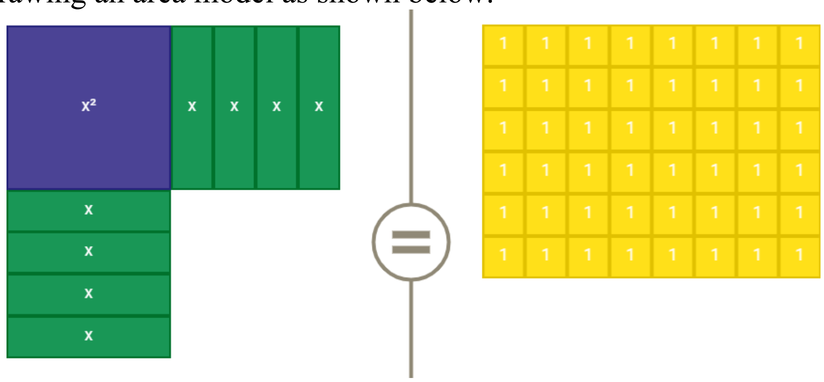
| Previous Benchmarks | Next Benchmarks |
| --- | --- |
| • MA.8.AR.2.3 | • MA.912.AR.3.2 |

### Purpose and Instructional Strategies

In grade 8, students solved quadratic equations in the form of $x^2 = p$. In Algebra I, students solve quadratic equations in one variable over the real number system. In later courses, students will solve quadratic equations in one variable over the real and complex number systems.

- Instruction includes the use of manipulatives, models, drawings and various methods, including Loh's method.
- Instruction allows the flexibility to solve quadratics using factoring techniques, taking the square root, using the quadratic formula and completing the square. Students should understand that one method may be more efficient than another depending on the content and context of the problem (MTR.2.1).
- Instruction emphasizes the understanding that solving a quadratic equation in one variable is the same as the process of determining $x$-intercepts, or roots, of the graph of a quadratic function.
- While the derivation of the quadratic formula is not an expectation of this benchmark, students can develop the quadratic formula by using completing the square to isolate $x$ in the equation $ax2 + bx + c = 0$; making the connection to solving literal equations.
- Instruction includes evaluating the discriminant ($b^2 - 4ac$) to determine whether there is one real solution (equals zero), two real solutions (equals a positive rational number) or two complex solutions (equals a negative rational number).

  - Discuss the connection of the number of solutions of a quadratic equation to the graph of a quadratic function. Guide students to see that real solutions result in roots or $x$-intercepts and that quadratic functions that do not produce real roots never touch the $x$-axis (MTR.5.1).
- Instruction on completing the square includes the use of algebra tiles or area model drawings. Students should understand the visual nature of completing the square and connect it to their prior work with area models.
  - For example, consider $x^2 + 8x = 48$ and solve by factoring to show that the two solutions are $x = -12$ and $x = 4$. Then, ask the question "If we look at the binomial $x^2 + 8x$ on the left, what would we have to add to it to make a perfect square trinomial?" Write the equation $x^2 + 8x + \quad = 48 + \quad$ on the board to represent the question. Now, represent the equation using algebra tiles or by drawing an area model as shown below.



  - Lead student discussion about the quantity of 1 unit by 1 unit tiles needed to "complete the square" on the left hand side. Once they state the need for 16 tiles, point out that 16 tiles must also be added to the right to maintain equivalence then write the number 16 into both blank boxes. Students can then factor the perfect square trinomial to arrive at $(x + 4)^2 = 64$. Have student solve using square roots to find the same solutions of $x = -12$ and $x = 4$ *(MTR.2.1, MTR.4.1, MTR.5.1)*.
- In many contexts, students may generate solutions that may not make sense when placed in context. Be sure students assess the reasonableness of their solutions in terms of context to check for this (MTR.6.1).
  - For example, the time it takes for a ball to drop from a height of 28 feet can be modeled by $0 = -16t^2 + 28$. Students solve this equation to find that $t \approx \pm 1.32$ seconds. Through discussion, students should see that $-1.32$ seconds does not make sense in context and therefore should be omitted.
- Enrichment of this benchmark includes determining if a quadratic is a perfect square trinomial.
  - For example, given the equation $0 = 4x^2 - 12x + 9$, students can identify $a$, $b$, and $c$ as 4, $-12$ and 9, respectively. Students should recognize that $a$ and $c$ are perfect squares; where a is $2^2 = 4$ and c is $(-3)^2 = 9$. Since the coefficient of $b$ is twice the product of the square roots $(-12 = 2(2)(-3))$, it can be determined that the given equation is a perfect square trinomial.

## Common Misconceptions or Errors

- When completing the square, many students forget to use the addition or subtraction property of equality to add or subtract values from the other side of the equation. Remind these students that additions or subtractions from one side of an equation must be replicated on the other to maintain equivalency.
- When completing the square and removing a common factor from the $x^2$ and $x$ term, students may forget to consider that factor when adding/subtracting from the other side of the equation.
  - For example, when solving $10 = 12t^2 + 48t + 55$, students may ultimately add 4 rather than 48 to the left side. Help students to see that there are 12 sets of 4 ultimately being added to the right and therefore, there must be twelve sets added to the left as well.
- Some students may see equations such as $x^2 - 4x + 6 = 27$ and use the number 2, $-4$ and 6 as $a$, $b$ and $c$, respectively, in the quadratic formula, arriving at incorrect solutions. In these cases, graph the related function and ask students if the solutions they calculated correspond to the roots of the parabola. Once they see they do not, have students set the equation equal to zero and recalculate.

## Strategies to Support Tiered Instruction

- When solving using the quadratic formula, instruction includes separating the two possible solutions to two equations.
  - For example, when determining the values of $x$ in the equation $6x^2 - 17x + 12 = 0$, students can set up the two equations using the quadratic formula: $x = \frac{-(-17)\pm\sqrt{(-17)^2-4(6)(12)}}{2(6)}$
- Teacher provides guided notes that show step-by-step directions for solving problems.
  - For example, teacher can use the directions below when using the quadratic formula.

| | |
|---|---|
| Determine the value(s) of $x$ in the equation. | $6x^2 - 17x = 12$ |
| Identify the $a$, $b$ and $c$ values in the equation. | $6x^2 - 17x - 12 = 0$<br>$a = 6, b = -17, c = -12$ |
| Substitute $a$, $b$ and $c$ into the quadratic formula. | $x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(6)(-12)}}{2(6)}$ |

- Teachers can remind students when completing the square that additions or subtractions from one side of an equation must be replicated on the other side to maintain equivalency.
- When addressing removing a common factor for completing the square, instruction includes highlighting the leading coefficient.
  - For example, when solving $10 = 12t^2 + 72t + 55$, students may add 9 rather than 108 to the left side in the completing square step. Help students to see the leading coefficient of 12 indicates there are 12 sets of 9 being added to the right and therefore, there must be twelve sets added to the left as well.
- Instruction should include equations with terms on both sides of the equation. Remind students to put the equation in standard form before using the quadratic formula.

- Teacher co-creates a graphic organizer, such as the one below, to include different methods used to solve quadratics given specific forms.

| Given Quadratic | Potential Method to Solve |
|---|---|
| Standard form where $a, b, c$ are large and or non-integer rational numbers | Quadratic Formula<br>Completing the Square<br>Loh's Method |
| Standard form where one set of factors for the product of $a$ and $c$ equal $b$<br><br>Two term quadratic with the $x^2$ and $x$ terms only | Factoring<br>Completing the Square |
| Perfect Square Trinomial | Factoring<br>Perfect Square Trinomial |
| Two term quadratic with the $x^2$ term and constant only<br><br>Vertex form | Taking Square Root |

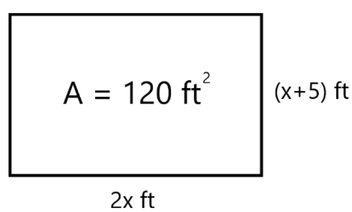
## Instructional Tasks

*Instructional Task 1 (MTR.2.1)*

Given the equation $x^2 + 6x = 13$, what value(s) of $x$ satisfy the equation?

*Instructional Task 2 (MTR.3.1)*

Given the figure below, write an equation that could be used to determine the length and width of the rectangle.




*Instructional Task 3 (MTR.3.1)*

Given the quadratic equation $x^2 + 6x = 13$.

a. Part A. Discuss the method or strategy you would use to find the solution(s). Explain why you chose that method.
b. Part B. How do you determine the number of solutions for the given equation? How do you know if the solutions are real or non-real?
c. Part C. Find the x-intercepts of the quadratic equation. What do the x-intercepts tell you about the equation's graph?
d. Part D. Work with a partner to create a real-world context for the equation.
e. Part E. How could you rewrite the given equation so that there is only one solution?

## Instructional Items

*Instructional Item 1*

Devonte throws a rock straight down off the edge of a cliff that overlooks the ocean. The distance ($d$) the rock falls after $t$ seconds can be represented by the equation $d = 16t^2 + 24t$. If the ocean's surface is 16.4 feet below the cliff, to the nearest tenth, how many seconds will it take for the rock to hit the ocean's surface?

*Instructional Item 2*

What are the solutions to the equation $-0.25x^2 + 4x = 0.75$. Round to the nearest tenth if necessary.

*Instructional Item 3*

What are the exact solutions to the equation $5x^2 - \frac{17}{2}x + \frac{3}{2}$?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.AR.3.4*

## Benchmark

**MA.912.AR.3.4** **Write a quadratic function to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.**

*Algebra I Example:* Given the table of values below from a quadratic function, write an equation of that function.

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | 2 | -1 | -2 | -1 | 2 |

Benchmark Clarifications:

*Clarification 1:* Within the Algebra I course, a graph, written description or table of values must include the vertex and two points that are equidistant from the vertex.

*Clarification 2:* Instruction includes the use of standard form, factored form and vertex form.

*Clarification 3:* Within the Algebra 2 course, one of the given points must be the vertex or an $x$-intercept.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.1
- MA.912.F.1.6
- MA.912.F.2.1

## Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Function Notation
- Quadratic Function
- Range
- $x$-intercept
- $y$-intercept

## Vertical Alignment

**Previous Benchmarks**
- MA.7.AR.4.3
- MA.8.AR.3.3

**Next Benchmarks**
- MA.7.AR.3.9

## Purpose and Instructional Strategies

In middle grades, students wrote two-variable linear equations in slope-intercept form. In Algebra I, students write quadratic functions from a graph, written description or table. In later courses, students will write quadratic two-variable inequalities.

- Instruction includes making connections to various forms of quadratic equations to show their equivalency. Students should understand when one form might be more useful than another depending on the context.
    - Standard Form
      Can be described by the equation $y = ax^2 + bx + c$, where $a$, $b$ and $c$ are any rational number. This form can be useful when identifying the $y$-intercept.
    - Factored Form
      Can be described by the equation $y = a(x - r_1)(x - r_2)$, where $r_1$ and $r_2$ are real numbers and the roots, or $x$-intercepts. This form can be useful when identifying the $x$-intercepts, or roots.
    - Vertex Form
      Can be described by the equation $y = a(x - h)^2 + k$, where the point $(h, k)$ is the vertex. This form can be useful when identifying the vertex, which is the turning point of the function where the axis of symmetry intersects.
- Instruction includes the use of $x$-$y$ notation and function notation.
- Instruction includes the connection to completing the square and literal equations to rewrite an equation from standard or factored form to vertex form.
- Instruction includes comparing and contrasting between a linear function of the form $y = a(x - h) + k$ and a quadratic function of the form $y = a(x - h)^2 + k$. This will also extend to an absolute value function of the form $y = a|x - h| + k$. *(MTR.5.1)*
- When determining the value of $a$ in a quadratic function, this can be done by two methods described below.
    - Students may notice a pattern from the points in the graph. When $a = 1$, points 1 unit to the left or right of the vertex are $1^2$ or 1 unit above or below the vertex. Points 2 units to the left or right of the vertex are $2^2$ or 4 units above or below the vertex. Students may look at the table and notice this relationship exists, therefore, $a = 1$.

        - This process can be used for other values of $a$. When $a = 2$, for example, points 1 unit to the left or right of the vertex are $2(1)^2$ or 2 units above or below the vertex. Points 2 units to the left or right of the vertex are $2(2^2)$ or 8 units above or below the vertex. Similarly, when $a = \frac{1}{2}$, points 1 unit to the left or right of the vertex are $\frac{1}{2}(1)^2$ or $\frac{1}{2}$ a unit above or below the vertex. Points 2 units to the left or right of the vertex are $\frac{1}{2}(2^2)$ or 2 units above or below the vertex.
    - Students can solve for $a$ by substituting the values of $x$ and $y$ from a point on the parabola.
        - Ask students to consider $y = a(x + 4)^2 - 2$ and determine what information they would need to be able to solve for $a$. As students express the need to know values for $x$ and for $y$, ask them if they know any combinations of $x$ and $y$ that are solutions. Students could use a table of values or a graph, if given, to determine values that could be used for $x$ and $y$. Have students pick one and substitute and solve for $a$. Ask for students who chose different points to share their value for $a$ to help them see that all points, when substituted, produce $a = 1$.
- Instruction includes the use of graphing software or technology.
    - For example, when determining the value of $a$, consider using graphing software to allow students to use sliders to quickly observe this. This provides opportunity for students to notice patterns regarding the value of $a$ and the concavity and stretch of the parabola (*MTR.5.1*).

## Common Misconceptions or Errors

- When writing functions in vertex form, students may confuse the sign of $h$.
    - For example, students may see a vertex of $(-1, -2)$ and an $a$ value of 3 and write the function as $y = 3(x - 1)^2 - 2$ instead of $y = 3(x + 1)^2 - 2$. To address this, help students recognize that because $h$ is subtracted from $x$ in vertex form, it will change the sign of that coordinate. Show students a graph of both functions to confirm and make the connection to transformation of functions (MA.912.F.2.1).

## Strategies to Support Tiered Instruction

- Teacher emphasizes that because $h$ is subtracted from $x$ in vertex form, it will change the sign of that coordinate. For example, students may see a vertex of $(-1, -2)$ and an $a$ value of 3 and write the function as $y = 3(x - 1)^2 - 2$ instead of $y = 3(x + 1)^2 - 2$. Teachers could show students a graph (technology) of both functions to confirm and make the connection to transformation of functions (MA.912.F.2.1).
- Teacher models substituting roots into the factored form of a quadratic, $y = a(x - r_1)(x - r_2)$. Instruction then includes reminding students of different methods that can be used to multiply binomials to convert the quadratic into standard form, $y = ax^2 + bx + c$.

- Instruction includes solving for $a$ by substituting the values of $x$ and $y$ from a point on the parabola. Students may need to review the order of operations to ensure they correctly isolate $a$. Provide students with a review of the order of operations.
  - Parenthesis
  - Exponents
  - Multiplication or Division (Whatever comes first left to right)
  - Addition or Subtraction (Whatever comes first left to right)
- Instruction includes recall of knowledge demonstrating when $h$ is subtracted from $x$ in vertex form, it will change the sign of that coordinate. A graph of both functions can be used to confirm and make the connection to transformation of functions.
- Teacher provides a laminated cue card of the steps required to convert from factored form to standard form.

## Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

Duane throws a tennis ball in the air. After 1 second, the height of the ball is 53 feet. After 2 seconds, the ball reaches a maximum height of 69 feet. After 3 seconds the height of the ball is 53 ft.

Part A. Write a quadratic function to represent the height of the ball, $h$, at any point in time, $t$.

Part B. How long will the tennis ball stay in the air? Round your answer to the nearest tenth second.

*Instructional Task 2 (MTR.4.1, MTR.5.1)*

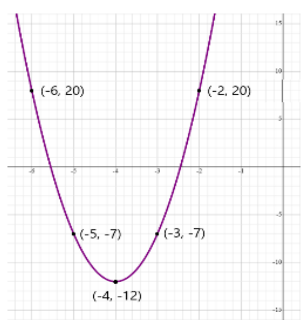
Part A. Create a quadratic function that contains the roots $\frac{3}{5}$ and 1.8.

Part B. Compare your function with a partner. What do you notice?

## Instructional Items

*Instructional Item 1*

Write a quadratic function to represent the graph below.




*Instructional Item 2*

Given the table of values below from a quadratic function, write an equation of that function.

| $x$ | -6 | -5 | -4 | -3 | -2 |
|---|---|---|---|---|---|
| $f(x)$ | 2 | -1 | -2 | -1 | 2 |

*Instructional Item 3*

Jayden launches a rocket from the ground that travels in a parabolic path until it lands. After two seconds it reaches a maximum height of 100 feet. The rocket is in the air for five seconds before it hits the ground. Write the quadratic function h(t), representing the height of the rocket above the ground after t seconds.

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**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

## *MA.912.AR.3.5*

### Benchmark

| | |
|---|---|
| **MA.912.AR.3.5** | **Given the $x$-intercepts and another point on the graph of a quadratic function, write the equation for the function.** |

### Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.3, MA.912.AR.1.7
- MA.912.F.1.6

### Terms from the K-12 Glossary

- Quadratic Function
- $x$-intercept
- $y$-intercept

### Vertical Alignment

**Previous Benchmarks**

- MA.8.AR.3.2

**Next Benchmarks**

- MA.912.AR.3.4

### Purpose and Instructional Strategies

In grade 8, students determined the slope in a linear relationship when given two points on the line. In Algebra I, students write the equation for a quadratic function when given the x-intercepts and another point on the graph. In later courses, students will write the equation for a quadratic function when given a vertex and another point on the graph.

- Instruction includes making connections to various forms of quadratic equations to show their equivalency. Students should understand and interpret when one form might be more useful than another depending on the context.
  - Standard Form
    Can be described by the equation $y = ax^2 + bx + c$, where $a$, $b$ and $c$ are any rational number. This form can be useful when identifying the $y$-intercept.
  - Factored form
    Can be described by the equation $y = a(x - r_1)(x - r_2)$, where $r_1$ and $r_2$ are real numbers and the roots, or $x$-intercepts. This form can be useful when identifying the $x$-intercepts, or roots.

- Vertex form
  Can be described by the equation $y = a(x - h)^2 + k$, where the point $(h, k)$ is the vertex. This form can be useful when identifying the vertex, which is the turning point of the function where the axis of symmetry intersects.
- Instruction includes comparing and contrasting between a linear function of the form $y = a(x - h) + k$ and a quadratic function of the form $y = a(x - h)^2 + k$. This will also extend to an absolute value function of the form $y = a|x - h| + k$. *(MTR.5.1)*
- Instruction includes the use of $x$-$y$ notation and function notation.
- Instruction includes the use of graphing technology.

## Common Misconceptions or Errors

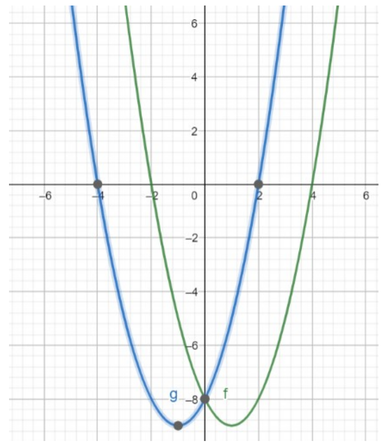
- Similar to their work with vertex form, students may have trouble with using the correct signs for $r_1$ and $r_2$ in factored form. In these cases, show students a corresponding graph of their developed factored form functions to help them see the need for opposite signs in their factors.
- Student might multiply incorrectly when squaring a binomial. For example, $(x - h)^2$ is incorrectly distributed as $x^2 - h^2$ instead of $x^2 - 2xh + h^2$.
- When rewriting their developed factored form functions into standard form, students may incorrectly multiply $a$ by both factors. One remedy for this is to direct students to multiply their binomials first and then multiply the product by $a$.

## Strategies to Support Tiered Instruction

- Teacher provides instruction to show how different methods can be used to multiply binomials after substituting the roots into factored form. Then, ask students to recall methods to multiply binomials. For example, direct students to multiply their binomials first and then multiply the product by $a$.
- Instruction includes converting a quadratic from vertex form to standard form. Instruct students to write out $(x - h)^2$ as $(x - h)(x - h)$. Remind students that different methods can be used to multiply binomials.
- Instruction includes an opportunity to discuss multiplicity of roots when converting from factored form to standard form. The teacher can use graphing software to build the connection between the standard form of a quadratic function with a perfect square trinomial and the resulting parabola with only one root. Students see that the root should be used for both $r_1$ and $r_2$ in factored form and confirm by converting their resulting function back to standard form.
  - For example, students could see a problem that only presents one root and express confusion on how it applies to the factored form. These cases provide an opportunity to discuss multiplicity of roots.
- Teacher models how to solve for $a$ by substituting the values of $x$ and $y$ from a point on the parabola.
- For students who need support evaluating functions, review the steps for evaluating a function given an input value by co-creating an anchor chart.

| Given Parabola: $y = 6x^2 - 5x + 11$ | |
|---|---|
| Input: $-4$ | Output |
| $y = 6(-4)^2 - 5(-4) + 11$ | $y = 127$ |

- Teacher provides opportunities to use an online graphing tool to graph an equation with the $x$-intercepts to help students who may have used the wrong signs in their factors.
  - For example, if the roots of a quadratic function are 2 and $-4$, the teacher can provide the graphs of the two functions (as shown below) to determine which equation corresponds to the roots.




Blue Graph: $y = (x - 2)(x + 4)$
Green graph: $y = (x + 2)(x - 4)$

## Instructional Tasks

*Instructional Task 1 ( MTR.7.1)*

A city fountain shoots jets of water that pass back and forth through a marble wall in its center. One jet of water begins 12 feet away from the wall and passes through a hole in the wall that is 12 feet high before landing 5 feet away on the other side. Write a quadratic function that represents the path the jet of water takes.

*Instructional Task 2 (MTR.3.1, MTR.6.1, MTR.7.1)*

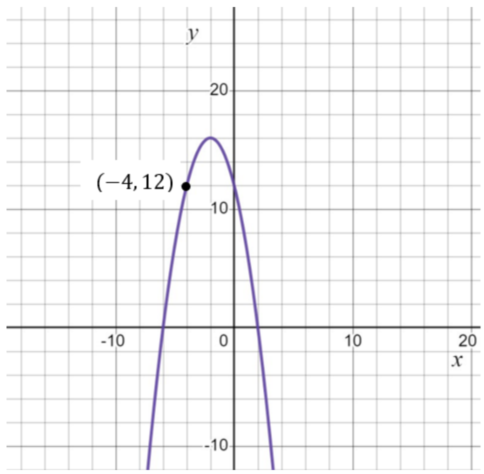
As part of a math project, you are analyzing the profit ($P$) of a smartphone manufacturing company based on the number of units sold ($x$). The company's profit can be modeled by a quadratic function. During your research, you discover that the company experiences a break-even point when no profit is made, and this occurs when no smartphones are sold. Additionally, when the company sells 1 smartphone, it makes a profit of \$6.

Write the quadratic equation that represents the company's profit, using the given information about the break-even point and the profit when selling one smartphone.

## Instructional Items

*Instructional Item 1*

Write a quadratic function to represent the graph below.




*Instructional Item 2*

Write a quadratic function to represent a parabola with roots of 5 and $-7$ that passes through the point $(-3, -128)$.

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**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.AR.3.6*

## Benchmark

| | |
|---|---|
| **MA.912.AR.3.6** | **Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context.** |

## Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.3, MA.912.AR.1.7
- MA.912.F.1.6

## Terms from the K-12 Glossary

- Quadratic Function
- $x$-intercept
- $y$-intercept

## Vertical Alignment

**Previous Benchmarks**

- MA.8.AR.1.2

**Next Benchmarks**

- MA.912.AR.6.4, MA.912.AR.6.5

**Purpose and Instructional Strategies**

In grade 8, students multiplied two linear expressions to obtain a quadratic expression. In Algebra I, students transform a quadratic function to highlight and interpret its vertex or its zeroes. In later courses, students will determine key features of higher degree polynomials.

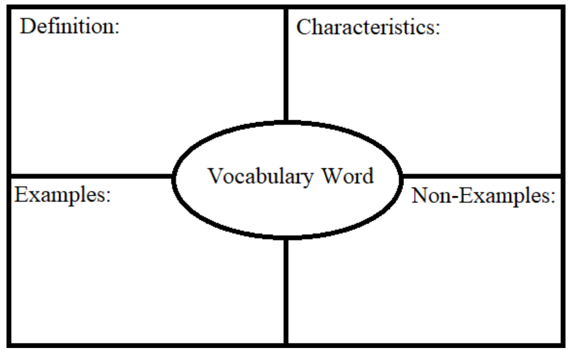
- Instruction includes making connections to various forms of quadratic equations to show their equivalency. Students should understand and interpret when one form might be more useful than another depending on the context.
  - Standard Form
    Can be described by the equation $y = ax^2 + bx + c$, where $a$, $b$ and $c$ are any rational number. This form can be useful when identifying the $y$-intercept.
  - Factored form
    Can be described by the equation $y = a(x - r_1)(x - r_2)$, where $r_1$ and $r_2$ are real numbers and the roots, or $x$-intercepts. This form can be useful when identifying the $x$-intercepts, or roots.
  - Vertex form
    Can be described by the equation $y = a(x - h)^2 + k$, where the point $(h, k)$ is the vertex. This form can be useful when identifying the vertex, which is the turning point of the function where the axis of symmetry intersects.
- Instruction includes comparing and contrasting between a linear function of the form $y = a(x - h) + k$ and a quadratic function of the form $y = a(x - h)^2 + k$. This will also extend to an absolute value function of the form $y = a|x - h| + k$. *(MTR.5.1)*
- Instruction includes the use of $x$-$y$ notation and function notation.
- Most contexts for this benchmark will present functions in standard form. Depending on their perspectives, students might take several approaches to determine vertices and zeros. Have students discuss strategies they might use. Let students know they should use the approach that is most efficient for them (*MTR.3.1*).
  - To determine zeros, students could use the quadratic formula or convert the function into factored form, or complete the square, or use Loh's method.
- To determine the vertex, students could convert the function into vertex form or determine the axis of symmetry $(x = \frac{-b}{2a})$. Calculate this value from one of the previous functions discussed and guide students to see that the vertex of each parabola falls on the line of symmetry. Considering this, they can substitute that $x$-value into the function to determine the corresponding $y$-value of the vertex.

**Common Misconceptions or Errors**

Some students may have difficulty interpreting the meaning of the zeros and vertex.

## Strategies to Support Tiered Instruction

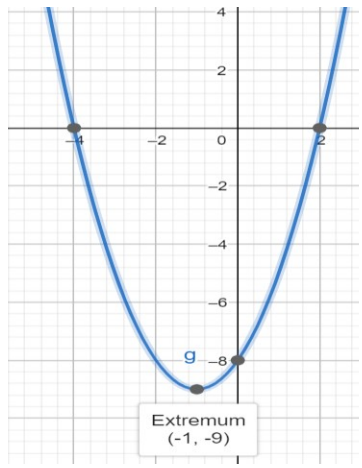
- Teacher provides a graphic organizer for key terms (zeros and vertex) that can be created using information provided in a given problem.




- Teacher co-creates a graphic organizer to compare real-world and mathematical contexts related to quadratics.
  - For example, the chart below can be used to compare mathematical terms with real-world context.

| $x$ | | $y$ | |
|---|---|---|---|
| **Mathematical Context** | **Real-world Context** | **Mathematical Context** | **Real-world Context** |
| Independent variable or input | Time | Dependent variable or output | Height |
| $x$-values, roots, solutions | Seconds for a ball to reach ground | $y$-value of vertex | Maximum height of a ball |

- Teacher provides a visual aid, including using graphing software, to interpret the zeros and vertex of a quadratic function. Students can compare definitions and examples to determine the zeros and vertex of the parabola.
  - For example, the roots of the graph shown are (2, 0) and (−4, 0). The locations on the graph that intersect the $x$-axis are the roots. The roots are also the solutions of the quadratic. The vertex of the graph shown is (−1, −9). The vertex is the minimum or maximum point (sometimes called the extrema) of the parabola. The vertex is also the point where the quadratic changes from decreasing to increasing or from increasing to decreasing, and is halfway between the roots.




## Instructional Tasks

*Instructional Task 1 (MTR.3.1, MTR.6.1, MTR.7.1)*

A diver jumps off a cliff 5 meters high into a lake. The diver's position can be represented by the function $h(t) = -4.9t^2 + 1.5t + 5$, where *h* represents the diver's height relative to the lake's surface and *t* represents the time in seconds.

Part A. Determine the roots of the function described.

Part B. Interpret each with respect to the situation.

*Instructional Task 2 (MTR.3.1, MTR.7.1)*

A local campground charges $23.50 per night per campsite. They average about 32 campsites rented each night. A recent survey indicated that for every $0.50 decrease, the number of campsites rented increases by five.

Part A. Write a quadratic equation that describes this situation.

Part B. Determine the vertex and zeroes of the function described.

Part C. Interpret each with respect to the situation.

Part D. What price will maximize revenue?

## Instructional Items

*Instructional Item 1*

Marcus just purchased a Super Bouncy Ball from a local toy store. Once outside, he throws it down to see how high the ball will bounce. The function $h(x) = -2x^2 + 24x - 31.5$ represents the height, $h$ in inches, above the ground of the ball as it relates to its horizontal distance from Marcus $x$, in inches. Find the zeros and vertex of this function and interpret the meaning of each.

*Instructional Item 2*

A hallway has an arch that is in the shape of a parabola. The arch is modeled by the function $y = -5x^2 + 12x$, where x represents the horizontal measurement, in feet, and y represents the vertical measurement, in feet. What are the zeroes of this function, and what do they represent?

A. The zeroes are (2.4,0) and (1.2, 7.2), which means these are the two places the archway touches the ground.
B. The zeroes are (0,0) and (2.4, 0), which means these are the two places the archway touches the ground.
C. The zeroes are (0,0) and (1.2, 7.2), which means that the maximum height of the arch is 7.2 feet.
D. The zeroes are (2.4, 0) and (1.2, 7.2), which means that the arch is 7.2 feet wide at the base.

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**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.AR.3.7*

## Benchmark

**MA.912.AR.3.7** **Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.**

Benchmark Clarifications:
*Clarification 1:* Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; vertex; and symmetry.
*Clarification 2:* Instruction includes the use of standard form, factored form and vertex form, and sketching a graph using the zeros and vertex.
*Clarification 3:* Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.
*Clarification 4:* Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.4.3
- MA.912.AR.5.6
- MA.912.F.1.6

## Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Function Notation
- Quadratic Function
- Range
- $x$-intercept
- $y$-intercept

## Vertical Alignment

**Previous Benchmarks**
- MA.8.AR.3.4

**Next Benchmarks**
- MA.912.AR.4.3, MA.912.AR.5.6, MA.912.AR.6.4

## Purpose and Instructional Strategies

In grade 8, students graphed linear two-variable equations given a table, written description or equation. In Algebra I, students graph quadratic functions given this same kind of information. In later courses, this work expands to other families of functions.

- Instruction includes conversations about interpreting $y$-intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and symmetry.
- When discussing end behavior, students should see a relationship between the sign of $a$ and the end behavior the function exhibits. Instruction presents students with the equation of a function first, before showing its graph and asking them to predict its end behavior (*MTR.5.1*).
  - Depending on the form the function is presented in, students may be able to predict other features as well (*MTR.5.1*).
- Instruction includes making connections to various forms of quadratic equations to show their equivalency. Students should understand and interpret when one form might be more useful than another depending on the context.
  - Standard Form
    Can be described by the equation $y = ax^2 + bx + c$, where $a$, $b$ and $c$ are any rational number. This form can be useful when identifying the $y$-intercept.
  - Factored form
    Can be described by the equation $y = a(x - r_1)(x - r_2)$, where $r_1$ and $r_2$ are real numbers and the roots, or $x$-intercepts. This form can be useful when identifying the $x$-intercepts, or roots.
  - Vertex form
    Can be described by the equation $y = a(x - h)^2 + k$, where the point $(h, k)$ is the vertex. This form can be useful when identifying the vertex, which is the turning point of the function where the axis of symmetry intersects.
- Instruction includes comparing and contrasting between a linear function of the form $y = a(x - h) + k$ and a quadratic function of the form $y = a(x - h)^2 + k$. This will also extend to an absolute value function of the form $y = a\,|x - h| + k$. *(MTR.5.1)*
- Instruction includes the use of $x$-$y$ notation and function notation.
- Instruction includes representing domain and range using words, inequality notation and set-builder notation.
  - Words
    If the domain is all real numbers, it can be written as "all real numbers" or "any value of $x$, such that $x$ is a real number."
  - Inequality notation
    If the domain is all values of $x$ greater than 2, it can be represented as $x > 2$.
  - Set-builder notation
    If the domain is all values of $x$ less than or equal to zero, it can be represented as $\{x | x \leq 0\}$ and is read as "all values of $x$ such that $x$ is less than or equal to zero."

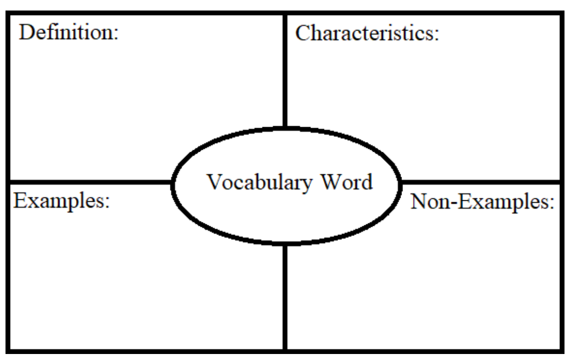
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the $x$- or $y$-axis when necessary.

## Common Misconceptions or Errors

- When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable.
- Students may miss the need for compound inequalities in their intervals. In these cases, refer to the graph of the function to help them discover areas in their interval that would not make sense in context.

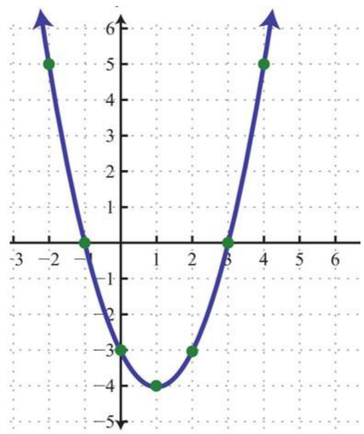
## Strategies to Support Tiered Instruction

- Teacher provides the opportunity to complete a graphic organizer to compare related key features.
  - For example, students can compare domain versus range, increasing versus decreasing or positive versus negative.
- Instruction includes modeling how to sketch the graph of the function to determine where the graph is increasing or decreasing. Students must understand that graphs are read left to right.
- Instruction includes reflective questions to examine the meaning of the domain and range in the problem.
  - For example, students can ask why the range does not contain all real numbers if the domain does contain all real numbers.
  - For example, students can ask how the vertex relates to the domain and range.
- Instruction references the graph of the function to help discover areas in compound inequality intervals that would not make sense in context.
- Teacher provides a graphic organizer for key features (vertex; symmetry; end behavior; intercepts) which can be completed using information provided in a given problem.



- Teacher provides a picture of a parabola (ensure the vertex and zeros are visible) and labels the parabola with a few of the key features. Then, teacher instructs students to label the parabola with the remaining key features.
    - For example, label the parabola with the coordinates of the vertex, axis of symmetry, minimum or maximum and zeros.

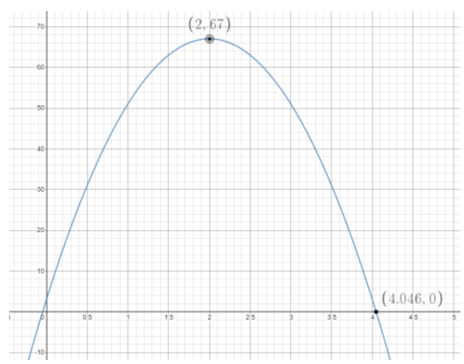
$$y = x^2 - 2x - 3$$



## Instructional Tasks

*Instructional Task 1 (MTR.3.1, MTR.7.1)*

A punter kicks a football to the opposing team. The trajectory of the football can be modeled by the function $h(t) = -16t^2 + 64t + 3$ where $h(t)$ represents the height of the football at any point in time (in seconds), $t$. A graph of the function is below.




Part A. Determine the meaning of the $y$-intercept in this context.
Part B. How high did the punt go?
Part C. Over what interval was the football increasing in height? When was the height decreasing?
Part D. What domain and range of this function is appropriate for the context?

*Instructional Task 2 (MTR.6.1, MTR.7.1)*

A company plans to build a large multiplex theater. The financial analyst told her manager that the profit function for their theater was $P(x) = -x^2 + 48x - 512$, where $x$ is the number of movie screens, and $P(x)$ is the profit earned in thousands of dollars.

Part A. Determine the range of production of movie screens that will guarantee that the company will not lose money.

Part B. What is the optimal number of movie screens the theater should have?

## Instructional Items

*Instructional Item 1*

Graph the function $f(x) = x^2 + 2x - 3$. Identify the domain, range, vertex and zeros of the function.

*Instructional Item 2*

Given the table of values, graph the function. Identify the domain, range, intervals where the function is increasing, decreasing, and the axis of symmetry.

| $x$ | -6 | -3 | 0 | 3 | 6 |
|---|---|---|---|---|---|
| $f(x)$ | -12 | -3 | 0 | -3 | -12 |

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.AR.3.8*

## Benchmark

**MA.912.AR.3.8** **Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.**

*Algebra I Example:* The value of a classic car produced in 1972 can be modeled by the function $V(t) = 19.25t^2 - 440t + 3500$, where $t$ is the number of years since 1972. In what year does the car's value start to increase?

Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; vertex; and symmetry.

*Clarification 2:* Instruction includes the use of standard form, factored form and vertex form.

*Clarification 3:* Instruction includes representing the domain, range and constraints with inequality notation, interval notation or set-builder notation.

*Clarification 4:* Within the Algebra I course, notations for domain, range and constraints are limited to inequality and set-builder.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.4.1
- MA.912.AR.5.3
- MA.912.F.1.6

## Terms from the K-12 Glossary

- Coordinate Plane
- Domain
- Function Notation
- Quadratic Function
- Range
- $x$-intercept
- $y$-intercept

## Vertical Alignment

**Previous Benchmarks**
- MA.8.AR.3.4

**Next Benchmarks**
- MA.912.AR5.6

## Purpose and Instructional Strategies

In grade 8, students solved problems involving real-world linear equations. In Algebra I, students solve problems that are modeled by quadratic functions. Students additionally graph the function and determine or interpret key features of the function. In later courses, this work expands to exponential and other kinds of functions.

- This benchmark is a culmination of MA.912.AR.3. Instruction here should feature a variety of real-world contexts. Some of these contexts should require students to create a function as a tool to determine requested information or should provide the graph or function that models the context.
- Instruction includes making connections to various forms of quadratic equations to show their equivalency. Students should understand and interpret when one form might be more useful than another depending on the context.
    - Standard Form
      Can be described by the equation $y = ax^2 + bx + c$, where $a$, $b$ and $c$ are any rational number. This form can be useful when identifying the $y$-intercepts.
    - Factored Form
      Can be described by the equation $y = a(x - r_1)(x - r_2)$, where $r_1$ and $r_2$ are real numbers and the roots, or $x$-intercepts. This form can be useful when identifying the $x$-intercepts, or roots.
    - Vertex Form
      Can be described by the equation $y = a(x - h)^2 + k$, where the point $(h, k)$ is the vertex. This form can be useful when identifying the vertex, which is the turning point of the function where the axis of symmetry intersects.
- Instruction includes comparing and contrasting between a linear function of the form $y = a(x - h) + k$ and a quadratic function of the form $y = a(x - h)^2 + k$. This will also extend to an absolute value function of the form $y = a\,|x - h| + k$. *(MTR.5.1)*
- Instruction includes the use of $x$-$y$ notation and function notation.
- Instruction includes representing domain, range and constraints using words, inequality notation and set-builder notation.
    - Words
      If the domain is all real numbers, it can be written as "all real numbers" or "any value of $x$, such that $x$ is a real number."

- Inequality Notation
  If the domain is all values of $x$ greater than 2, it can be represented as $x > 2$.
- Set-Builder Notation
  If the domain is all values of $x$ less than or equal to zero, it can be represented as $\{x|x \leq 0\}$ and is read as "all values of $x$ such that $x$ is less than or equal to zero."

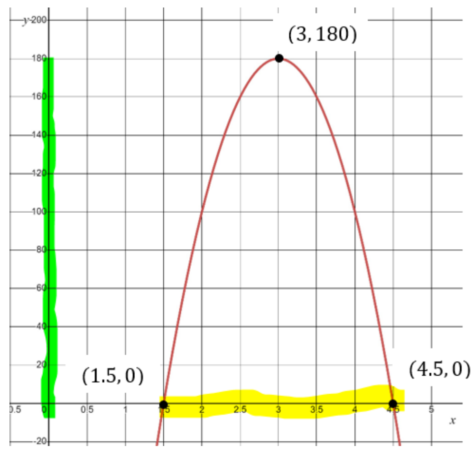
- Instruction provides opportunities to make connections between the domain and range and other key features.
  - For example, a coffee shop uses the function, $P(x) = -80x^2 + 480x - 540$ to model the profit they can earn in thousands of dollars in terms of the price per cup of coffee, in dollars. By determining the domain and range that includes prices that yield a positive profit, one would also have to identify the vertex (or maximum) and the roots of the function. Students should realize that they can do this by transforming the given expression into vertex form.
  - Marcus just purchased a Super Bouncy Ball from a local toy store. Once outside, he throws it down to see how high the ball will bounce. The function $h(x) = -2x^2 + 24x - 31.5$ represents the height, $h$ in inches, above the ground of the ball as it relates to its horizontal distance from Marcus $x$, in inches. In order to determine domain and range that includes positive heights, one would identify the vertex (or maximum) and the roots of the function. Students should realize that they can do this by transforming the given expression into vertex form.

## Common Misconceptions or Errors

- Students might not understand the connection between x-y notation and function notation.

## Strategies to Support Tiered Instruction

- Teacher provides equations in both function notation and $x$-$y$ notation and models graphing both forms using a graphing tool or graphing software *(MTR.2.1)*.
  - For example, $f(x) = (x - 2.3)^2 + 7$ and y $= (x - 2.3)^2 + 7$, to show that both $f(x)$ and $y$ represent the same outputs of the function.
- Instruction provides opportunities to visualize the domain and range on a graph using a highlighter.
  - For example, a coffee shop uses the function $P(x) = -80x^2 + 480x - 540$ to model the profit they can earn in thousands of dollars in terms of the price per cup of coffee, in dollars. If the coffee shop is only interested in prices that yield a positive profit, the highlighted domain and range are shown




## Instructional Tasks

*Instructional Task 1 (MTR.4.1, MTR.912.1, MTR.7.1)*

ABC Pool Company is constructing 17 feet by 11 feet rectangular pool. Along each side of the pool, they plan to construct a concrete sidewalk that has a constant width. The land parcel being used has a total area of 315 sq. ft. to construct the pool and sidewalks. The function$f(x) = 4x^2 + 56x - 128$, represents the situation described. Transform the function to determine and interpret its zeros.

## Instructional Items

*Instructional Item 1*

A new coffee shop wants to maximize their profit within the first year of business. They determined the function, $P(x) = -80x^2 + 480x - 540$, models the profit they can earn in thousands of dollars in terms of the price per cup of coffee, in dollars. What price of coffee maximizes the coffee shop's profit?

*Instructional Item 2*

The value of a classic car produced in 1972 can be modeled by the function $V(t) = 19.25t^2 - 440t + 3500$, where $t$ is the number of years since 1972.

Part A. In what year does the car's value start to increase?
Part B. After the year found in Part A, will the car's value ever begin to decrease? Assume it follows its modeled value.

*Instructional Item 3*

A startup tech company aims to optimize its quarterly revenue during the initial year of operation. They have identified the function $R(x) = -2x^2 + 150x - 200$, representing the revenue they can generate in thousands of dollars based on the price per software license, denoted in dollars. Graph the function and determine at what price per software license maximizes the company's quarterly revenue?

*The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

**MA.912.AR.5** *Write, solve and graph exponential and logarithmic equations and functions in one and two variables.*

*MA.912.AR.5.3*

## Benchmark

| | |
|---|---|
| **MA.912.AR.5.3** | **Given a mathematical or real-world context, classify an exponential function as representing growth or decay.** |

Benchmark Clarifications:
*Clarification 1:* Within the Algebra I course, exponential functions are limited to the forms $f(x) = ab^x$, where $b$ is a whole number greater than 1 or a unit fraction, or $f(x) = a(1 \pm r)^x$, where $0 < r < 1$.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1
- MA.912.AR.1.1
- MA.912.F.1.6, MA.912.F.1.8

## Terms from the K-12 Glossary

- Exponential Function

## Vertical Alignment

**Previous Benchmarks**
- MA.7.AR.3

**Next Benchmarks**
- MA.912.AR.5.5

## Purpose and Instructional Strategies

In middle grades, students solved problems involving percentages, including percent increases and decreases. In Algebra I, students identify and describe exponential functions in terms of growth or decay rates. In later courses, students will further develop their understanding of exponential functions and how they are characterized by having a constant percent of change per unit interval.

- Provide opportunities to reference MA.912.AR.1.1 as students identify and interpret parts of an exponential equation or expression as growth or decay.
- Instruction includes the connection to growth or decay of a function as a key feature (constant percent rate of change) of an exponential function and being useful in understanding the relationships between two quantities.
- Instruction includes the use of graphing technology to explore exponential functions.
  - For example, students can explore the function $f(x) = ab^x$ and how the $a$-value and $b$-value are affected. Ask questions like "What impact does changing the value of $a$ have on the graph? What about $b$? What values for $b$ cause the function to increase? Which values cause it to decrease?" As students explore, formalize the terms exponential growth and decay when appropriate.
  - As students explore the graph, have students choose values of $a$ and $b$ to

complete a table of values. Once completed ask students what causes the value of $y$ to increase or decrease as the value of $x$ increases. Guide students to see that it's because $b > 1$ or $b < 1$. Have students adjust the graph and repeat this exercise.
  - Once students have an understanding of what causes exponential growth and decay, both graphically and algebraically, ask students if they think the curve ever passes $y = 0$. Have students extend their function table for exponential decay to include more extreme values for $x$ to explore if it ever does. As students arrive at an understanding that it does not cross $y = 0$, guide them to understand why it doesn't algebraically. Once students arrive at this understanding, define this boundary as an *asymptote*.
  - As students explore the provided graph, they will move the slider for $b$ to have negative values. The resulting graphs provide an interesting discussion point. Have students complete a function table using negative $b$ values. Students should quickly see the connection between the two "curves" and why neither is continuous. Let students know that for this reason, most contexts for exponential functions restrict $b$ to be greater than 0 and not equal to 1.
- As students solidify their understanding of $f(x) = ab^x$, use graphing technology again to have them explore the form $f(x) = a(1 \pm r)^x$. Guide students to use the sliders for $a$ and $r$ to visualize that only $r$ determines whether the function represents exponential growth or exponential decay.
  - Have students discuss which values for $r$ cause exponential growth or decay. They should observe that negative values cause exponential decay while positive values cause exponential growth.

## Common Misconceptions or Errors

- Students may not understand exponential function values will eventually get larger than those of any other polynomial functions because they do not fully understand the impact of exponents on a value.
- Students may not understand that growth factors have one constraint ($b > 1$) while decay factors have a compound constraint ($0 < b < 1$). Some students may think that as long as $b < 1$, the function will represent exponential decay.
- Students may think that if $a$ is negative and $r > 0$ or $b > 1$, the function represents an exponential decay. To address this misconception, help students understand that the negative values are growing at an exponential rate.

## Strategies to Support Tiered Instruction

- Teacher provides instruction to identify exponential functions in all methods (i.e., graphs, equations and tables).
  - For example, instruction may include providing a comparison of the two forms of exponential functions. Having a side-by-side comparison of both as an equation, graph and a table of values will provide a visual aid.
- Teacher provides student with examples and non-examples of exponential functions in tables to help students visualize the growth or decay of a function.
  - For example, teacher can provide the following tables for students to determine which ones represent an exponential function.

| $x$ | $y$ |
|---|---|
| 4 | 6 |
| 5 | 36 |
| 6 | 216 |
| 7 | 1296 |
| 8 | 7776 |

| $x$ | $y$ |
|---|---|
| $-1$ | 2 |
| $-2$ | 6 |
| $-3$ | 18 |
| $-4$ | 54 |
| $-5$ | 162 |

| $x$ | $y$ |
|---|---|
| 0 | 10 |
| 1 | 17 |
| 2 | 24 |
| 3 | 31 |
| 4 | 38 |

## Instructional Tasks

*Instructional Task 1 (MTR.4.1, MTR.7.1)*

After a person takes medicine, the amount of drug left in the person's body changes over time. When testing a new drug, a pharmaceutical company develops a mathematical model to quantify this relationship. To find such a model, suppose a dose of 1000 mg of a certain drug is absorbed by a person's bloodstream. Blood samples are taken every five hours, and the amount of drug remaining in the body is calculated. The data collected from a particular sample is recorded below.

| **Drug Absorption Data** | |
|---|---|
| Hours Since Drug was Administered | Amount of Drug in Body (mg) |
| 0 | 1000 |
| 5 | 550 |
| 10 | 316 |
| 15 | 180 |
| 20 | 85 |
| 25 | 56 |
| 30 | 31 |

Part A. Does this data represent an exponential growth or decay function? Justify your answer.

Part B. Create an exponential function that describes the data in the table above.

*Instructional Task 2 (MTR.3.1, MTR.5.1)*

Given the function $h(x) = 10^{0.2x}$, what is the rate of growth or decay?

## Instructional Items

*Instructional Item 1*

Given the function $f(x) = 125(1 - 0.26)^x$, does it represent an exponential growth or decay function?

*Instructional Item 2*

Dunia is fascinated by the growth of bacteria in a controlled laboratory environment. At the beginning of the experiment, she observes that there are 10 bacteria. The population of bacteria increase by 100% every hour. Which statement describes the population of the bacteria over time?

a. The population of bacteria represents a decay of two every hour.
b. The population of bacteria represents growth of 10 every hour.
c. The population of bacteria represents growth by doubling every hour.
d. The population of bacteria represents decay by halving every hour.

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**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

## *MA.912.AR.5.4*

### Benchmark

| | |
|---|---|
| **MA.912.AR.5.4** | **Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.** |

Benchmark Clarifications:

*Clarification 1:* Within the Algebra I course, exponential functions are limited to the forms $f(x) = ab^x$, where $b$ is a whole number greater than 1 or a unit fraction, or $f(x) = a(1 \pm r)^x$, where $0 < r < 1$.

*Clarification 2:* Within the Algebra I course, tables are limited to having successive nonnegative integer inputs so that the function may be determined by finding ratios between successive outputs.

### Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1
- MA.912.AR.1.1
- MA.912.F.1.6, MA.912.F.1.8

### Terms from the K-12 Glossary

- Exponential

### Vertical Alignment

**Previous Benchmarks**

- MA.7.AR.3

**Next Benchmarks**

- MA.912.AR.5.5, MA.912.AR.5.7

## Purpose and Instructional Strategies

In middle grades, students solved problems involving percentages, including percent increases and decreases and write equations that represent proportional relationships. In Algebra I, students write exponential functions that model relationships characterized by having a constant percent of change per unit interval. In later courses, students will further develop their understanding of this feature of exponential functions.

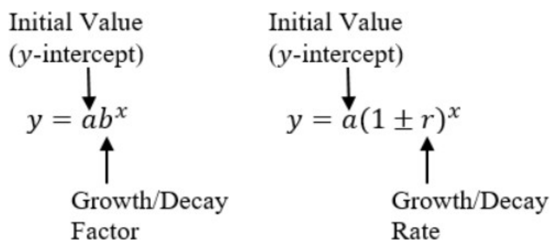
- Provide opportunities to reference MA.912.AR.1.1 as students identify and interpret parts of an exponential equation or expression as growth or decay and connect them to key features of the graph.
- Problems include cases where the initial value is not given.
- Instruction includes guidance on how to determine the initial value or the percent rate of change of an exponential function when it is not provided.
    - For example, if the initial value of $(0,3)$ is given, students can now write the function as $f(x) = 3b^x$. Guide students to choose a point on the curve that has integer coordinates such as $(2, 12)$. Lead them to substitute the point into their function to find $b$. Students should recognize that exponential functions are restricted to positive values of $b$, leading to the function $f(x) = 3(2)^x$.
- Instruction includes interpreting percentages of growth/decay from exponential functions expressed in the form $f(x) = ab^x$ and see that $b$ can be used to determine a percentage.
    - For example, the function $f(x) = 500(1.16)^x$ represents 16% growth of an initial value.
        - Guide students to discuss the meaning of the number 1.16 as a percent. They should understand it represents 116%. Taking 116% of an initial value increases the magnitude of that value. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to exponential growth.
    - For example, the function $f(x) = 500(0.72)^x$ represents 28% decay of an initial value.
        - Guide students to discuss the meaning of the number 0.72 as a percent. They should understand it represents 72%. Taking 72% of an initial value decreases the magnitude of that value. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to exponential decay.
    - For example, the function $f(x) = 500(1)^x$ represents an initial value that neither grows nor decays as $x$ increases.
        - Guide students to discuss the meaning of the number 1 when it comes to growth/decay factors. They should understand it represents 100%. Taking 100% of an initial value causes the value to remain the same. (Students can test this in a calculator to confirm.) Taking this percentage repetitively leads to no change in the initial value (explaining the horizontal line that shows when $b = 1$ on the graph).

## Common Misconceptions or Errors

- Students may not understand that exponential function values will eventually get larger than those of any other polynomial functions because they do not fully understand the impact of exponents on a value.

## Strategies to Support Tiered Instruction

- Teacher provides students with a graphic displaying key terms within an exponential function.




- Instruction includes comparing quadratic (polynomial) to exponential functions using graphs or tables with whole-number inputs to show how exponential functions will quickly exceed quadratic (polynomial) functions.

## Instructional Tasks

*Instructional Task 1 (MTR.4.1, MTR.5.1, MTR.7.1)*

Karl and Simone were working on separate biology experiments. Each student documented their cell population counts over time in the chart below.

| **Karl's Experiment** | |
|---|---|
| # of minutes | # of cells |
| 5 | 2800 |
| 10 | 1400 |
| 15 | 700 |
| 20 | 350 |
| 25 | 175 |

| **Simone's Experiment** | |
|---|---|
| # of minutes | # of cells |
| 0 | 80 |
| 1 | 120 |
| 2 | 180 |
| 3 | 270 |
| 4 | 405 |

Part A. Do the number of cells in Simone's experiment increase at a constant percentage rate of change? If so, what is the percentage rate? If not, describe what is happening to the number of cells. Does this change represent growth or decay? Justify your answer.

Part B. Write exponential functions to represent the relationship between the quantities for each student's experiment. In which experiment are the number of cells changing more rapidly? Justify your answer.

Part C. Graph these functions and determine their key features.

*Instructional Task 2 (MTR.2.1, MTR.7.1)*

The population of J-Town in 2019 was estimated to be 76,500 people with an annual rate of increase of 2.4%.

Part A. Write an equation to model future growth.

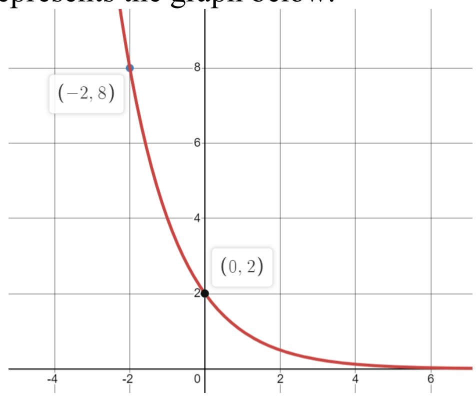
Part B. What is the growth factor for J-Town?

Part C. Use the equation to estimate the population in 2072 to the nearest hundred people.

## Instructional Items

*Instructional Item 1*

Write an exponential function that represents the graph below.




*Instructional Item 2*

A forester has determined that the number of fir trees in a forest is decreasing by 3% per year. In 2010, there were 13,000 fir trees in the forest. Write an equation that represents the number of fir trees, *N*, in terms of *t,* the number of years since 2010.

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**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.AR.5.6*

## Benchmark

| | |
|---|---|
| **MA.912.AR.5.6** | **Given a table, equation or written description of an exponential function, graph that function and determine its key features.** |

Benchmark Clarifications:

*Clarification 1:* Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; constant percent rate of change; end behavior and asymptotes.

*Clarification 2:* Instruction includes representing the domain and range with inequality notation, interval notation or set-builder notation.

*Clarification 3:* Within the Algebra I course, notations for domain and range are limited to inequality and set-builder.

*Clarification 4:* Within the Algebra I course, exponential functions are limited to the forms $f(x) = ab^x$, where $b$ is a whole number greater than 1 or a unit fraction or $f(x) = a(1 \pm r)^x$, where $0 < r < 1$.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.1.1
- MA.912.F.1.6, MA.912.F.1.8

## Terms from the K-12 Glossary

- Coordinate plane
- Domain
- Exponential Function
- Function Notation
- Range
- $x$-intercept
- $y$-intercept

## Vertical Alignment

**Previous Benchmarks**
- MA.8.AR.3

**Next Benchmarks**
- MA.912.AR.8.2

## Purpose and Instructional Strategies

In grade 8, students graphed linear equations. In Algebra I, students graph exponential functions and determine their key features, including asymptotes and end behavior. Students are first introduced to asymptotes in Algebra I. In later courses, asymptotes are important in the study of other types of functions, including rational functions.

- Instruction includes defining an asymptote as a line that a curve approaches but does not meet as it heads toward positive or negative infinity.
- Instruction provides the opportunity for students to explore the meaning of an asymptote graphically and algebraically. Through work in this benchmark, students will discover asymptotes are useful guides to complete the graph of a function, especially when drawing them by hand. For mastery of this benchmark, asymptotes can be drawn on the graph as a dotted line or not drawn on the graph.
- For students to have a full understanding of exponential functions, instruction includes MA.912.AR.5.3 and MA.912.AR.5.4. Growth or decay of a function can be defined as a key feature (constant percent rate of change) of an exponential function and useful in understanding the relationships between two.
- Instruction includes the use of $x$-$y$ notation and function notation.
- Instruction includes representing domain and range using words, inequality notation and set-builder notation.
    - Words
      If the domain is all real numbers, it can be written as “all real numbers” or “any value of $x$, such that $x$ is a real number.”
    - Inequality notation
      If the domain is all values of $x$ greater than 2, it can be represented as $x > 2$.
    - Set-builder notation
      If the domain is all values of $x$ less than or equal to zero, it can be represented as $\{x|x \leq 0\}$ and is read as “all values of $x$ such that $x$ is less than or equal to zero.”
- Instruction includes the use of appropriately scaled coordinate planes, including the use of breaks in the $x$- or $y$-axis when necessary.

## Common Misconceptions or Errors

- Students may not fully understand how to use proper notation when determining the key features of an exponential function.

## Strategies to Support Tiered Instruction

- Instruction includes student understanding that growth and decay is not the same as a function increasing or decreasing.
  - For example, the exponential function $y = -2(0.5)^x$ is an exponential decay function because the value of $b$ is in between 0 and 1. Note that it is the magnitude of the $y$-values that are decaying exponentially, eventually getting closer to zero. However, the value of the function increases as the value of $x$ increases. To help students visualize this, graph the function using graphing technology.
- Instruction includes using an exponential function formula guide like the one below.

| Exponential Growth | Exponential Decay |
|---|---|
| $b > 1$ | $b < 1$ |
| $y = a(1 + r)^t$ | $y = a(1 - r)^t$ |

## Instructional Tasks

*Instructional Task 1*

The bracket system for the NCAA Basketball Tournament (also known as March Madness) is an example of an exponential function. At each round of the tournament, teams play against one another with only the winning teams progressing to the next round. If we start with 64 teams going into round 1, the table of values looks something like this:

| | | | | | |
|---|---|---|---|---|---|
| Round | 1 | 2 | 3 | 4 | 5 |
| Number of teams left playing at end of round | 32 | 16 | 8 | 4 | 2 |

Part A. Graph this function.
Part B. What is the percentage of teams left after each round?

*Instructional Task 2*

Ashanti purchased a car for \$22,900. The car depreciated at an annual rate of 16%. After 5 years, Ashanti wants to sell her car.

Part A. Write an equation that models the value of Ashanti's car?
Part B. What would be the range of the graph of the value of Ashanti's car?
Part C. What would be the $y$-intercept of that graph and what does it represent?
Part D. Will her car ever have a value of \$0.00 based on your equation?
Part E. What is a reasonable domain for this function? Justify your answer.

## Instructional Items

*Instructional Item 1*

An exponential function is given by the equation y=-14. What is the asymptote for the graph?

*Instructional Item 2*

An exponential function is given.

$$y = 50(1.1)^t$$

Part A. Does this function represent exponential growth or decay?

Part B. What is the constant percent rate of change of $y$ with respect to $t$.

*Instructional Item 3*

Graph the exponential function represented by the table of values. Identify the domain, range, constant percent rate of change and end behavior.

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**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

**MA.912.AR.9** *Write and solve a system of two- and three-variable equations and inequalities that describe quantities or relationships.*

*MA.912.AR.9.6*

## Benchmark

| | |
|---|---|
| **MA.912.AR.9.6** | **Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.** |

Benchmark Clarifications:
*Clarification 1:* Instruction focuses on analyzing a given function that models a real-world situation and writing constraints that are represented as linear equations or linear inequalities.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.2, MA.912.AR.2.5, MA.912.AR.2.7, MA.912.AR.2.8
- MA.912.AR.3.8

## Terms from the K-12 Glossary

- Inequality
- Linear Equation

## Vertical Alignment

**Previous Benchmarks**
- MA.8.AR.2.2
- MA.8.AR.4.1, MA.8.AR.4.2, MA.8.AR.4.3

**Next Benchmarks**
- MA.912.AR.4.4
- MA.912.AR.5.7, MA.912.AR.5.9
- MA.912.AR.6.6
- MA.912.AR.7.3, MA.912.AR.7.4
- MA.912.AR.8.3
- MA.912.AR.9.7, MA.912.AR.9.10
- MA.912.T.3.3

## Purpose and Instructional Strategies

in grade 8, students worked with linear equations and inequalities, and graphically solved systems of linear equations. In Algebra I, students represent constraints as systems of linear equations or inequalities and interpret solutions as viable or non-viable options. In later courses, students will solve problems involving linear programming and work with constraints within various function types.

- For students to have a full understanding of systems, instruction includes MA.912.AR.9.1 and MA.912.AR.9.4. Equations and inequalities and their constraints are all related and the connections between them should be reinforced throughout the instruction.
- Allow for both inequalities and equations as constraints. Include cases where students must determine a valid model of a function.
  - Students often use inequalities to represent constraints throughout Algebra I. Equations can be thought of as constraints as well. Solving a system of equations requires students to find a point that is constrained to lie on specific lines simultaneously.

- Instruction includes the use of various forms of linear equations and inequalities.
  - Standard Form
    Can be described by the equation $Ax + By = C$, where $A$, $B$ and $C$ are any rational number and any equal or inequality symbol can be used.
  - Slope-intercept form
    Can be described by the inequality $y \geq mx + b$, where $m$ is the slope and $b$ is the $y$-intercept and any equal or inequality symbol can be used.
  - Point-slope form
    Can be described by the inequality $y - y_1 > m(x - x_1)$, where $(x_1, y_1)$ is a point on the line and $m$ is the slope of the line and any equal or inequality symbol can be used.

## Common Misconceptions or Errors

- Students may have difficulty translating word problems into systems of equations and inequalities.
- Students may shade the wrong half-plane for an inequality.
- Students may graph an incorrect boundary line (dashed versus solid) due to incorrect translation of the word problem.
- Students may not identify the restrictions on the domain and range of the graphs in a system of equations based on the context of the situation.

## Strategies to Support Tiered Instruction

- Instruction provides opportunities to translate systems of equations or inequalities from word problems by first creating equations, then by identifying keywords to determine the inequality symbol (i.e., no more than, less than, at least, etc.). The appropriate inequality symbols can then replace the equal signs. Students can separate and organize information for each equation or inequality.
  - Separate given information for each equation or inequality.
  - Determine the appropriate form of equation or inequality based on givens.
  - Define a variable to represent the item wanted in the equation or inequality.
  - Determine what values are constants or should be placed with the variables.
  - Write the equation or inequality.
- Teacher co-creates a graphic organizer to scaffold graphing an inequality and its solution. The steps can be repeated for each inequality.

| ***Symbol (s)*** | ***Graph*** |
|---|---|
| $<$ and $>$ | Dashed line,<br>Solution does not include points on the line |
| $\leq$ and $\geq$ | Solid line,<br>Solution includes points on the line |

- Instruction includes opportunities to identify a test point to substitute into an inequality to determine which symbol should be used when writing the inequality. It is usually easiest to use the origin $(0,0)$ as it makes mental calculations easier. If the point selected creates a true statement, the half plane that includes the test point should be shaded. If it creates a false statement, the half plane that does not include the test point should be shaded.
- Teacher makes connections back to students' understanding of MA.912.AR.2.5 and

MA.912.AR.3.8 and writing constraints based on a real-world context.

- For example, Dani is planning her wedding, and the venue charges a flat rate of $8250 for four hours. The venue can provide meals for each of the guests and charges $21.25 per plate for adults and $13.75 per plate for children if she has a minimum of 75 guests. If Dani's budget is $38,000, students can describe this situation using the inequalities $a + c \geq 75$ and $21.25a + 13.75c + 8250 \leq 38000$. Depending on the number of adults and children she wants to invite and the capacity of the venue, students can determine various other constraints.

- Teacher provides questions to be answered by students to aid in the identification of domain and range restrictions:
  - Does the problem involve humans, animals or things that cannot or are normally not broken into parts? If yes, you are restricted by integers.
  - Do negative numbers not make sense? If yes, you are restricted by positive numbers.
  - Was a maximum or minimum value given? If yes, the solution must not exceed the maximum or drop lower than the minimum.
- Instruction includes identifying which variable(s) the constraints apply to.

## Instructional Tasks

*Instructional Task 1 (MTR.3.1)*

A baker has 16 eggs and 15 cups of flour. One batch of chocolate chip cookies requires 4 eggs and 3 cups of flour. One batch of oatmeal raisin cookies requires 2 eggs and 3 cups of flour. The baker makes $3 profit for each batch of chocolate chip cookies and $2 profit for each batch of oatmeal raisin cookies. How many batches of each cookie should she make to maximize profit?

*Instructional Task 2 (MTR.4.1)*

Amy and Anthony are starting a pet sitter business. To make sure they have enough time to properly care for the animals, they create a feeding and pampering plan. Anthony can spend up to 8 hours a day taking care of the feeding and cleaning, and Amy can spend up to 8 hours each day pampering the pets.

*Feeding/Cleaning Time:* Amy and Anthony estimate they need to allot 6 minutes twice a day, morning and evening, to feed and clean litter boxes for each cat, a total of 12 minutes a day per cat. Dogs will require 10 minutes twice a day to feed and walk, for a total of 20 minutes per day for each dog.

*Pampering Time:* Sixteen minutes per day will be allotted for brushing and petting each cat and 20 minutes each day for bathing and playing with each dog.

Part A. Write an inequality for feeding/cleaning time needed for the pets. Represent all time in the same unit (minutes or hours).

Part B. Write an inequality for pampering time needed for the pets. Represent all time in the same unit (minutes or hours).

Part C. Graph the two inequalities.

Part D. In term of this scenario, explain the meanings of the following points: $(0,24)$ and $(30,0)$.

Part E. What is the greatest number of dogs they can watch if they are watching 19 cats?

Part F. List two viable combinations of pets that can be watched.

Possibility 1: ________cats _________dogs

Possibility 2: ________cats __________dogs

## Instructional Items

*Instructional Item 1*

There are several elevators in the Sandy Beach Hotel. Each elevator can hold at most 12 people. Additionally, each elevator can only carry 1600 pounds of people and baggage for safety reasons. Assume on average an adult weighs 175 pounds and a child weighs 70 pounds. Also assume each group will have 150 pounds of baggage plus 10 additional pounds of personal items per person.

Part A. Write a system of linear equations or inequalities that describes the weight limit for one group of adults and children on a Sandy Beach Hotel elevator and that represents the total number of passengers in a Sandy Beach Hotel elevator.

Part B. Several groups of people want to share the same elevator. Group 1 has 4 adults and 3 children. Group 2 has 1 adult and 11 children. Group 3 has 9 adults. Which of the groups, if any, can safely travel in a Sandy Beach elevator?

*Instructional Item 2*

A farmer is planning to plant two types of crops, Crop X and Crop Y. The maximum amount of field space is 6 acres, and the maximum water supply is 10 units. Each acre of Crop X requires 1 unit of water, and each acre of Crop Y requires 2 units of water. Write a system of equations or inequalities that represents the situation.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

# Functions

**MA.912.F.1** *Understand, compare and analyze properties of functions.*

*MA.912.F.1.1*

## Benchmark

| | |
|---|---|
| **MA.912.F.1.1** | **Given an equation or graph that defines a function, classify the function type. Given an input-output table, determine a function type that could represent it.** |

Benchmark Clarifications:
*Clarification 1:* Within the Algebra I course, functions represented as tables are limited to linear, quadratic and exponential.
*Clarification 2:* Within the Algebra I course, functions represented as equations or graphs are limited to vertical or horizontal translations or reflections over the $x$-axis of the following parent functions:

$f(x) = x$, $f(x) = x^2$, $f(x) = x^3$, $f(x) = \sqrt{x}$, $f(x) = \sqrt[3]{x}$, $f(x) = |x|$, $f(x) = 2^x$ and $f(x) = \left(\frac{1}{2}\right)^x$

## Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2
- MA.912.AR.3
- MA.912.AR.4
- MA.912.DP.2.6

## Terms from the K-12 Glossary

- Exponential Function
- Function
- Linear Function
- Quadratic Function

## Vertical Alignment

**Previous Benchmarks**
- MA.8.F.1.1, MA.8.F.1.2, MA.8.F.1.3

**Next Benchmarks**
- MA.912.AR.5
- MA.912.AR.6
- MA.912.AR.7
- MA.912.AR.8
- MA.912.GR.7
- MA.912.T.2

## Purpose and Instructional Strategies

In grade 8, students identified the domain and range of a relation and determined whether it is a function or not. In Algebra I, students classify function types limited to simple linear, quadratic, cubic, square root, cube root, absolute value and exponential functions. In later courses, students will classify other function types.

- The purpose of this benchmark is to lay the groundwork for students to be able to choose appropriate functions to model real-world data.

- Instruction includes the connection of the graph to its parent function. See Clarification 1 for specifics of the Algebra I course.
- Students will work extensively with linear, quadratic and exponential models in the Algebra I course. Strong attention should be given to the other function types so that students can build familiarity with them. As new function types are introduced, take time to allow students to produce a rough graph of the parent function from a table of values they develop. Lead student discussion to build connections with why these function types produce their corresponding graphs (*MTR.4.1*).
- Instruction develops the understanding that if given a table of values, unless stated, one cannot absolutely determine the function type, but state which function the table of values could represent.
    - For example, if given the function $y = |x|$ and only positive values were given in a table, one could say that table of values could represent a linear or absolute value function.

## Common Misconceptions or Errors

- Some students may miscalculate first and second differences that deal with negative values, especially if they perform them mentally.

## Strategies to Support Tiered Instruction

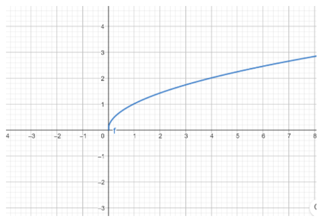
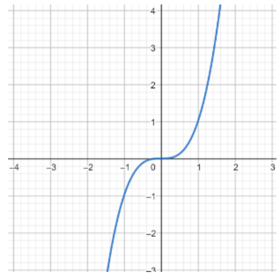
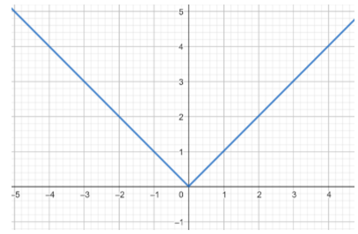
- Teacher provides opportunities to write out subtraction sentences next to each line of the table when determining first and second differences.
- Instructions are provided to determine the type of function the graph represents. Knowledge on the end behavior of different types of functions may provide students with additional information to identify different types of functions. The teacher co-creates an anchor chart showing different types of functions and their end behavior.
- Teacher provides methods for calculating and/or interpreting the first and second differences given a table of values.

| Exponential | | |
|---|---|---|
| −3 | 2 | × 2 |
| −2 | 4 | × 2 |
| −1 | 8 | × 2 |
| 0 | 16 | × 2 |
| 1 | 32 | |

| Linear | | |
|---|---|---|
| 3 | −20 | +3 |
| 4 | −17 | +3 |
| 5 | −14 | +3 |
| 6 | −11 | +3 |
| 7 | −8 | |

| Quadratic | | | |
|---|---|---|---|
| 1 | 7 | | |
| 2 | 13 | +6 | |
| 3 | 23 | +10 | +4 |
| 4 | 37 | +14 | +4 |
| 5 | 55 | +18 | +4 |

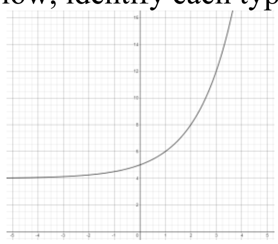
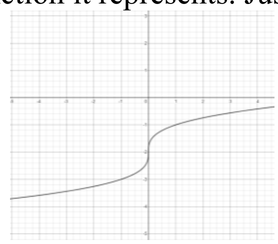
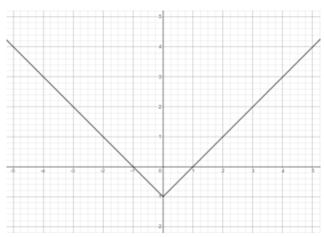
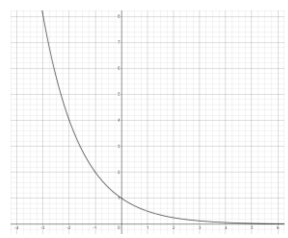
- Instruction includes opportunities to use graphing software to graph parent functions of different equations (i.e., square root, cubic, absolute value, etc.).

| $y = \sqrt{x}$ | $y = x^3$ | $y = \lvert x \rvert$ |
|---|---|---|
|  |  |  |

## Instructional Tasks

*Instructional Task 1 (MTR.3.1)*

Given the graphs below, identify each type of function it represents. Justify your answer.

## Instructional Items

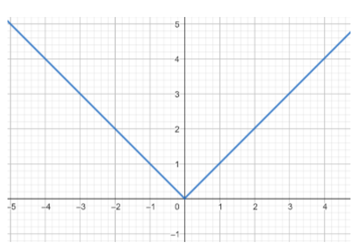
*Instructional Item 1*

Given the table below, determine the function type that could represent it.

| $x$ | 6 | 8 | 10 | 12 | 14 |
|---|---|---|---|---|---|
| $y$ | -1.5 | 0 | 2.5 | 6 | 10.5 |

*Instructional Item 2*

Determine the function type of the graph below.



*Instructional Item 3*

Determine the function type of the equation, $f(x) = 5x + 2$.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

## *MA.912.F.1.2*

| | **Benchmark** |
|---|---|
| **MA.912.F.1.2** | **Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.** |

Benchmark Clarifications:
*Clarification 1:* Problems include simple functions in two-variables, such as $f(x, y) = 3x - 2y$.
*Clarification 2:* Within the Algebra I course, functions are limited to one-variable such as $f(x) = 3x$.

### Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2
- MA.912.AR.3.8
- MA.912.AR.4.3
- MA.912.AR.5.6
- MA.912.F.3.1

### Terms from the K-12 Glossary

- Function Notation

### Vertical Alignment

| Previous Benchmarks | Next Benchmarks |
|---|---|
| • MA.6.AR.1.2 | • MA.912. F.3 |

### Purpose and Instructional Strategies

In middle grades, students worked with $x$-$y$ notation and substituted values in expressions and equations. In Algebra I, students work with $x$-$y$ notation and function notation throughout instruction of linear, quadratic, exponential and absolute value functions. In later courses, students will continue to use function notation with other function types and perform operations that combine functions, including compositions of functions.

- Instruction leads students to understand that $f(x)$ reads as "$f$ of $x$" and represents an output of a function equivalent to that of the variable $y$ in $x$-$y$ notation.
- Instruction includes a series of functions with random inputs so that students can see the pattern that emerges (*MTR.5.1*).
    - For example,

$$f(x) = 2x^2 + 5x - 7$$
$$f(k) = 2k^2 + 5k - 7$$
$$f(-2) = 2(-2)^2 + 5(-2) - 7$$

- Students should discover that the number in parenthesis corresponds to the input or x-value on the graph and the number to the right of the equal sign corresponds to the output or y-value.

- Although not conventional, instruction includes using function notation flexibly.
  - For example, function notation can be written as $h(\text{x}) = 4x + 7$ or $4x + 7 = h(x)$.
- Instruction leads students to consider the practicality that function notation presents to mathematicians. In several contexts, multiple functions can exist that we want to consider simultaneously. If each of these functions is written in $x$-$y$ notation, it can lead to confusion in discussions.
  - For example, representing the equations, $y = -2x + 4$ and $y = 3x + 7$, in function notation allows mathematicians to distinguish them from each other more easily (i.e., $f(x) = -2x + 4$ and $g(x) = 3x + 7$).
- Function notation also allows for the use of different symbols for the variables, which can add meaning to the function.
  - For example, $h(t) = -16t^2 + 49t + 4$ could be used to represent the height, $h$, of a ball in feet over time, $t$, in seconds.
- Function notation allows mathematicians to express the output and input of a function simultaneously.
  - For example, $h\ (3) = 7$ would represent a ball 7 feet in the air after 3 seconds of elapsed time. This is equivalent to the ordered pair $(3, 7)$ but with the added benefit of knowing which function it is associated with.

## Common Misconceptions or Errors

- Throughout students' prior experience, two variables written next to one another indicate they are being multiplied. These changes in function notation and will likely cause confusion for some of your students.
- Students may need additional support in the order of operations.
  - For example, for exponential functions, many students multiply $a$ by the growth factor and then raise the product to the value of the exponent.
  - For example, students may think that multiplication is always performed before division.

## Strategies to Support Tiered Instruction

- Instruction includes discussing the meaning of function notation with students with understanding that $f(x)$ does not mean $f \cdot x$.
- Instruction is provided to determine the order of operations required once a given input is placed into the function for evaluation. Students may need additional support determining the correct order of operations to perform.
- Teacher models using parentheses to help organize order of operations when evaluating functions.
  - When evaluating $f(x) = 4x^2$ for $x = -1$, teacher can model the use of parentheses by writing the expression $4(-1)^2$ rather than without using parentheses writing $4 \cdot -1^2$. This will help students visualize the operations.

- Teacher provides instruction for identifying the operations in various functions as they relate to the order of operations using a graphic organizer.

| Find $f(5)$ | $f(x) = 4x^3 - 3.5x^2 + 10$ |
|---|---|
| Substitute 5 for $x$. | $f(5) = 4(5)^3 - 3.5(5)^2 + 10$ |
| Evaluate the exponents. | $f(5) = 4(125) - 3.5(25) + 10$ |
| Multiply the factors within each term. | $f(5) = 500 - 87.5 + 10$ |
| Perform addition and subtraction of terms from left to right. | $f(5) = 412.5 + 10$<br>$f(5) = 422.5$ |

## Instructional Tasks

*Instructional Task 1 (MTR.3.1)*

The original value of a painting is $9,000 and the value increases by 7% each year. The value of the painting can be described by the function $V(t) = 9000(1 + 0.07)^t$, where $t$ is the time in years since 1984 and $V(t)$ is the value of the painting.

Part A. Create a table of values that corresponds to this function.

Part B. Graph the function.

## Instructional Items

*Instructional Item 1*

Evaluate $f(24)$, when $f(x) = \frac{3}{2}x + 9$.

*Instructional Item 2*

Given $f(x) = 2x^4 - 0.24x^2 + 6.17x - 7$, find $f(2)$.

*Instructional Item 3*

A shipping company charges a base fee of $20 plus an additional $5 for each package shipped. The total cost $(C)$ of shipping p packages can be modeled by the function $C(p) = 20 + 5p$ where $p$ is the number of packages. You are a small business owner looking to send out 10 packages to customers. Evaluate the function $C$ for $p = 10$ to find the total cost of shipping the packages.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.F.1.3*

## Benchmark

| MA.912.F.1.3 | **Calculate and interpret the average rate of change of a real-world situation represented graphically, algebraically or in a table over a specified interval.** |
|---|---|

Benchmark Clarifications:
*Clarification 1:* Instruction includes making the connection to determining the slope of a particular line segment.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.2
- MA.912.FL.3.4

## Terms from the K-12 Glossary

- Rate of Change
- Slope (of a graph)

## Vertical Alignment

**Previous Benchmarks**
- MA.8.AR.3.2
- MA.8.F.1.3

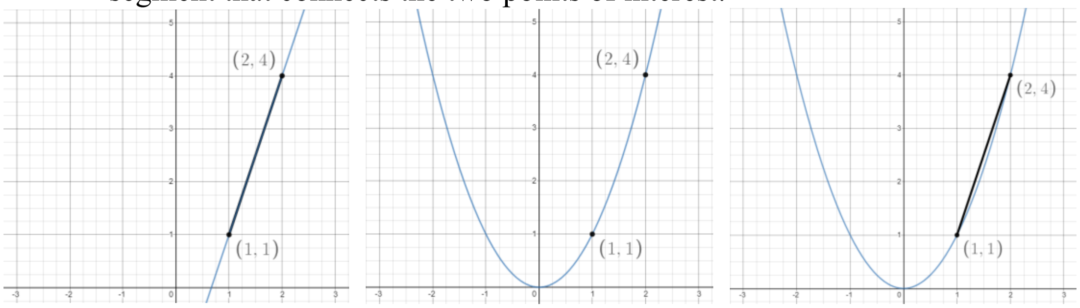
**Next Benchmarks**
- MA.912.F.1.4
- MA.912.C.3.8, MA.912.C.3.9, MA.912.C.3.10

## Purpose and Instructional Strategies

In grade 8, students determined the slope, constant rate of change, of a linear equation in two variables and analyzed graphical representations of functional relationships. In Algebra I, students calculate the average rate of change in real-world situations represented in various ways. In later courses, this concept leads to the difference quotient and differential calculus.

- The purpose of this benchmark is to extend students' understanding of the rate of change to allow them to apply it in non-linear contexts.
- Instruction emphasizes a graphical context so students can see the meaning of the average rate of change. Students can use graphing technology to help visualize this.
  - Starting with the linear function $f(x) = 3x - 2$, shown below, ask students to calculate the rate of change between two points using the slope formula. Lead students to verify their calculations visually.
  - Once students have successfully used the formula, transition to the graph of $f(x) = x^2$.

- Highlight the same two points and ask students to discuss what the rate of change might be between them (*MTR.4.1*). Lead students to realize that while there is not a constant rate of change, they can calculate an *average* rate of change for an interval. Show students that this is equivalent to calculating the slope of the line segment that connects the two points of interest.




- Once students have an understanding, ask them to find the average rate of change for other intervals, such as $-2 \leq x \leq -1$ or $0 \leq x \leq 2$. As each of these calculations produce different values, reinforce the concept that non-linear functions do not have constant rates of change (*MTR.5.1*).
- Look for opportunities to continue students' work with function notation. Ask students to find the average rate of change between $f(1)$ and $f(4)$ for $f(x)$.

## Common Misconceptions or Errors

- Some graphs presented to students will only display a certain interval of data. Some students may mistakenly interpret the rate of change for that entire interval rather than a given sub-interval.
- Students may confuse the average rate of change and constant rate of change.
- Students may be confused if the average rate of change is 0, even though the function is not constant.
  - For example, the average rate of change of the function $f(x) = x^2$ from $x = -1$ to $x = 1$ is 0.

## Strategies to Support Tiered Instruction

- When determining an average rate of change on an interval where the function only increases or only decreases, instruction includes directions to highlight the domain and range for the interval so that the change can be seen more clearly.
- Instruction includes providing the graph to visualize the change in the x- and $y$-values by drawing a line from the leftmost point on the graph within the interval to the rightmost point on the graph within the interval. Discuss with students how the average rate of change is the slope of the line between the two points.
  - For example, for the function $f(x) = x^2$, on the interval $[-2,2]$, the average rate of change is zero. This can be visualized by drawing a line from the point $(-2,4)$ to $(2,4)$ which has slope of zero (horizontal line).
- Instruction includes assistance recognizing the connection between slope (MA.912.AR.2) and rate of change for a linear function. A linear function has a constant rate of change. Regardless of the interval, the rate of change is the same (constant). For nonlinear functions rate of change is not constant, so it is not considered slope. However, the formula for slope can be used to calculate the average rate of change over an interval.
- Teacher provides instruction on assigning the ordered pairs when calculating average rate

of change. The formula for slope is the change in $y$ divided by the change in $x$. The designation of the interval points as the first or second pair of coordinates does not matter.

- Teacher co-creates an $x$-$y$ chart to organize information when solving real-world problems involving a graphical representation. The $x$-column should be labeled with the input description used for the $x$-axis. The $y$-column should be labeled with the output description used for the $y$-axis.

## Instructional Tasks

*Instructional Task 1 (MTR.3.1, MTR.4.1)*

Jorge invests an amount of $5,000 in a money market account at an annual interest rate of 5%, compounded monthly. The function $f(x) = 5000\left(1+\frac{0.05}{12}\right)^{12x}$ represents the value of the investment after $x$ years.

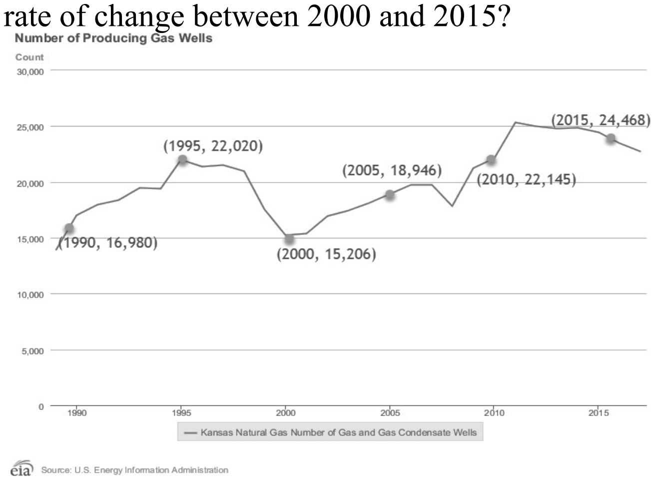
Part A. Find the average rate of change in the value of Jorge's investment between year 5 and year 7, between year 10 and year 12, and between year 15 and year 17.

Part B. What do you notice about the change in value of the investment over each interval?

## Instructional Items

*Instructional Item 1*

The graph below represents the number of producing gas wells in Kansas from 1989 to 2017. What is the average rate of change between 2000 and 2015?



*Instructional Item 2*
A student is taking a science test. The table shows the number of questions they have remaining and the time that has passed.

| Time (minutes) | Remaining questions |
|---|---|
| 0 | 30 |
| 10 | 25 |
| 20 | 18 |
| 30 | 12 |
| 40 | 5 |
| 50 | 0 |

What is the average rate of change, to the nearest tenth, from 10 minutes to 40 minutes, and what does it mean?

A. On average, the student answered 0.7 questions per minute during the first 30 minutes.
B. On average, the student answered 1.5 questions per minute during the first 30 minutes.
C. On average, the student answered 0.7 questions per minute during those 30 minutes.
D. On average, the student answered 1.5 questions per minute during those 30 minutes.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

## *MA.912.F.1.6*

### Benchmark

| MA.912.F.1.6 | **Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.** |
|---|---|

Benchmark Clarifications:
*Clarification 1:* Key features are limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior and asymptotes.
*Clarification 2:* Within the Algebra I course, functions other than linear, quadratic or exponential must be represented graphically.
*Clarification 3:* Within the Algebra I course, instruction includes verifying that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

### Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4, MA.912.AR.2.5
- MA.912.AR.3.7, MA.912.AR.3.8
- MA.912.AR.4.6
- MA.912.AR.5.6

## Terms from the K-12 Glossary

- Domain
- Intercept
- Range
- Slope

## Vertical Alignment

**Previous Benchmarks**
- MA.8.AR.3.5

**Next Benchmarks**
- MA.912.F.1.7

## Purpose and Instructional Strategies

In grade 8, students interpreted the slope and $y$-intercept of a linear equation in two variables. In Algebra I, students compare key features of two or more linear or nonlinear functions. Except for quadratic and exponential functions, nonlinear functions must be represented graphically. In later courses, students will compare key features of nonlinear functions represented graphically, algebraically, or with written descriptions.

- Within this benchmark, one of the functions given must be linear and the number of functions being compared is not limited to two.
- Problem types include comparing linear to nonlinear functions represented graphically and also opportunities that present linear, quadratic and exponential functions in different forms.
- Instruction includes student exploration of linear, quadratic and exponential models to ultimately determine that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.
    - For example, provide the following context.
        - You are being contracted by a large company to provide technical services to a major engineering project. The contract will involve you advising a group of engineers for the three weeks. The company offers you a choice of two methods of payment for your services. The first is to receive \$500 per day of work. The second is to receive payment on a scale: two cents for one total day of work, four cents for two total days, eight cents for three total days, etc. Which method of payment would you choose?

      Have students choose a method of payment and begin a class discussion regarding the reasoning students used to make their choices (*MTR.4.1*). Ask if students can create a function to represent each payment method (*MTR.7.1*).
- Instruction includes representing domain and range using words, inequality notation and set-builder notation.
    - Words
      If the domain is all real numbers, it can be written as “all real numbers” or “any value of $x$, such that $x$ is a real number.”
    - Inequality notation
      If the domain is all values of $x$ greater than 2, it can be represented as $x > 2$.
    - Set-builder notation
      If the domain is all values of $x$ less than or equal to zero, it can be represented as $\{x|x \leq 0\}$ and is read as “all values of $x$ such that $x$ is less than or equal to zero.”

## Common Misconceptions or Errors

- When describing domain or range, students may assign their constraints to the incorrect variable.
- Students may miss the need for compound inequalities when describing domain or range. When describing intervals where functions are increasing, decreasing, positive or negative, students may represent their interval using the incorrect variable.

## Strategies to Support Tiered Instruction

- Teacher provides a laminated cue card to aid in the identification of domain and range restrictions:
  - Is the constraint on the independent or dependent variable in the context of the problem?
  - Does the constraint restrict the input or output value in the context of the problem?
  - Was the constraint shown or highlighted on the $x$- or $y$-axis?
- Teacher provides a chart to show different terminology associated with domain and range.

| ***Domain*** | ***Range*** |
|---|---|
| $x$-values | $y$-values |
| Input | Output |

- The use of a graph of the function to point out areas of constraints in a real-world context can help students understand the need for compound inequalities when describing the domain and range.
- Where students are struggling with concepts such as when a function is increasing, decreasing, positive, negative or questions about its end behavior, ask reflective questions:
  - Imagine walking on the graph from left to right, where would you be going uphill (increasing) or where would you be going downhill (decreasing)?
  - On the left, where are you coming from (far below or far above), and on the right, would you eventually be going up forever or down forever (end behavior)?

## Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

Nancy works for a company that offers two types of savings plans. Plan A is represented by the function $g(x) = 250 + 3x$, where $x$ is the number of quarter years she has utilized the plan. Plan B is represented by the function $(x) = 250\ (1.01)\ x$, where $x$ is the number of quarter years she has utilized the plan.

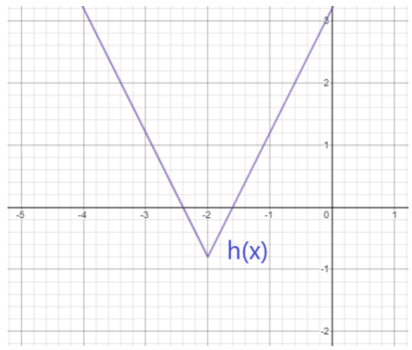
Part A. Nancy wants to have the highest savings possible after five years, when she plans to leave the company. Which plan should she use?

Part B. What if Nancy stays for ten years? Which plan should she use?

*Instructional Task 2 (MTR.3.1)*

Three functions are represented below, with the table representing a linear function. Which function has the smallest x-intercept?

$f(x) = 25x^2 - 16$



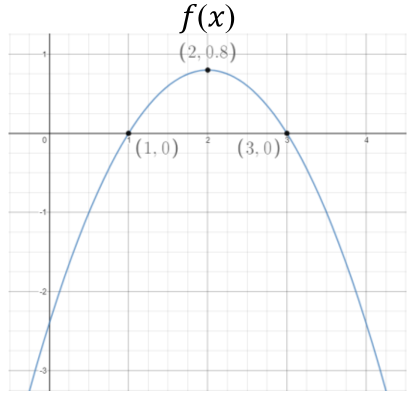

| $x$ | -5 | -2 | 1 | 4 | 7 |
|---|---|---|---|---|---|
| $g(x)$ | -8.6 | -2 | 4.6 | 11.2 | 17.8 |

## Instructional Items

*Instructional Item 1*

The functions $f(x)$ and $g(x)$ are shown below, with $g(x)$ representing a linear function. Which function has the greater $y$-intercept?

$f(x)$




| $x$ | 3 | 7 | 11 | 15 | 19 |
|---|---|---|---|---|---|
| $g(x)$ | -1.1 | 0.5 | 2.1 | 3.7 | 5.3 |

*Instructional Item 2*

The function f(x) can be represented by $f(x) = 3(1.06)^x$. The function g(x) has a slope of 7 and a y-intercept of -2. As x increases, which function will eventually exceed the y values of the other function and why?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.F.1.8*

## Benchmark

**MA.912.F.1.8** **Determine whether a linear, quadratic or exponential function best models a given real-world situation.**

Benchmark Clarifications:
*Clarification 1:* Instruction includes recognizing that linear functions model situations in which a quantity changes by a constant amount per unit interval; that quadratic functions model situations in which a quantity increases to a maximum, then begins to decrease or a quantity decreases to a minimum, then begins to increase; and that exponential functions model situations in which a quantity grows or decays by a constant percent per unit interval.
*Clarification 2:* Within this benchmark, the expectation is to identify the type of function from a written description or table.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.F.1.1, MA.912.DP.2.4

## Terms from the K-12 Glossary

- Exponential Function
- Linear Function
- Quadratic Function

## Vertical Alignment

**Previous Benchmarks**
- MA.8.AR.3.1

**Next Benchmarks**
- MA.912.DP.2.8
- MA.912.DP.2.9

## Purpose and Instructional Strategies

In grade 8, students determined whether a linear relationship is also a proportional relationship. In Algebra I, students determine whether a linear, quadratic or exponential function best models a situation. In later grades, students will fit linear, quadratic and exponential functions to statistical data.

- Instruction should include identifying function types from tables and from written descriptions.
  - When examining written descriptions, guide students to see that linear functions model situations in which a quantity changes by a constant amount per unit interval; that quadratic functions model situations in which a quantity increases to a maximum, then begins to decrease or a quantity decreases to a minimum, then begins to increase; and that exponential functions model situations in which a quantity grows or decays by a constant percent per unit interval.
  - When considering tables, instruction guides students to understand that linear relationships have a common difference per unit interval (or a constant rate of change), quadratic relationships produce a common second difference, and exponential relationships produce a constant percent rate of change per unit interval (*MTR.5.1*).
  - Considering tables like the one below, lead students to discover that there is a common difference of 0.4 between successive $y$-values. Plotting these points using graphing software will verify that they are collinear.

| $x$ | -4 | -3 | -2 | -1 | 0 |
|---|---|---|---|---|---|
| $y$ | 1.6 | 2.0 | 2.4 | 2.8 | 3.2 |
| 1st Difference | 0.4 | 0.4 | 0.4 | 0.4 | |

- Considering tables like the one below, lead students to discover that there is a common second difference of -8 between successive $y$-values. Plotting these points using graphing software will verify that they form a parabola. It is important to note that using the common second difference method is not an expectation for mastery in Algebra I, as students will expand on this in Algebra 2.

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | -14 | -2 | 2 | -2 | -14 |
| 1st Difference | 12 | 4 | -4 | -12 | |
| 2nd Difference | -8 | -8 | -8 | | |

- Considering tables like the one below, lead students to discover that there is no common difference or second difference. In this case, there is a common ratio of between successive $y$-values. Plotting these points using graphing software will verify that they form an exponential graph.

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $y$ | 3 | 9 | 27 | 81 | 243 |
| 1st Difference | 6 | 18 | 54 | 162 | |
| 2nd Difference | 12 | 36 | 108 | | |
| Common Ratio | $\frac{3}{1}$ | $\frac{3}{1}$ | $\frac{3}{1}$ | $\frac{3}{1}$ | |

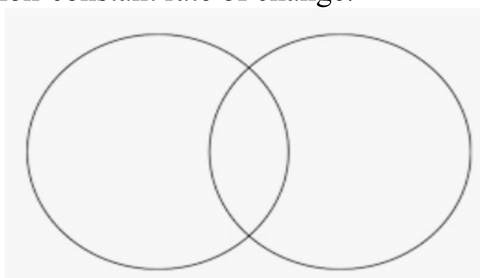
- Students should note that the search for common differences and ratios only works when the $x$-values are equidistant from each other. Lead them to check for this when presented with tables of values to consider.
- It is important to note that other function types could produce these relationships, making the connection to classifying different function types in MA.912.F.1.1.

## Common Misconceptions or Errors

- Students may interpret any relationship that increases/decreases at a non-constant rate as being an exponential relationship.
  - Some students may miscalculate first and second differences that deal with negative values, especially if they perform them mentally.

## Strategies to Support Tiered Instruction

- Instruction includes verifying an exponential relationship by looking for common ratios. When interpreting a written description, make a sample table of values from the context to examine the type of function.
- Teacher co-creates a graphic organizer to compare exponential and quadratic functions.
  - For example, a Venn Diagram can be used with the common middle section including the non-constant rate of change.



- Teacher provides opportunities to write out subtraction sentences next to each line of the table when determining first and second differences. Have students write out the subtraction expression [i.e., $-14 - (-2)$] so they can see that they are subtracting a negative value and should convert it to adding a positive value.
  - It is often helpful to have these students draw a blank number line with a mark for 0 to use for their calculations. Students who solve $-14 + 2$ to equal $-16$ could place their pencil tip to the left of 0 on the number line in a position that could represent $-14$. Ask them which direction they would move to represent adding 2. When students see movement to the right, toward zero, they should understand that the magnitude of the negative number decreases, resulting in $-12$ rather than $-16$.

## Instructional Tasks

*Instructional Task 1 (MTR.3.1)*

A scientist is monitoring cell division and notes that a single cell divides into 4 cells within one hour. During the next hour, each of these cells divides into 4 cells. This process continues at the same rate every hour.

Part A. What type of function could be used to represent this situation?

Part B. Justify your reasoning.

## Instructional Items

*Instructional Item 1*

Sarah is spending the summer at her grandmother's house. The table below shows the amount of money in her bank account at the end of each week. What type of function could be used to model the total amount of money in Sarah's bank account as a function of time?

| Week # | Total $ |
|---|---|
| 1 | $3428 |
| 2 | $3276 |
| 3 | $3124 |
| 4 | $2972 |

*Instructional Item 2*

A rectangular garden is being expanded by planting additional rows of flowers along one of its sides. The length of the garden is currently 10 feet, and each week it is increased by 3 feet. Which type of function best models the situation?

a. Linear
b. Quadratic
c. Exponential
d. None of the above

*Instructional Item 3*

An investment grows over time with a fixed annual interest rate. An individual invests $1,000 in a savings account, and each year the investment grows by 5%. Identify the type of function that best models the situation.

---

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

**MA.912.F.2** *Identify and describe the effects of transformations on functions. Create new functions given transformations.*

*MA.912.F.2.1*

### Benchmark

| | |
|---|---|
| **MA.912.F.2.1** | **Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ for specific values of $k$.** |

Benchmark Clarifications:
*Clarification 1:* Within the Algebra I course, functions are limited to linear, quadratic and absolute value.
*Clarification 2:* Instruction focuses on including positive and negative values for $k$.

### Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.4
- MA.912.AR.3.7
- MA.912.AR.4.3
- MA.912. F.1.1

### Terms from the K-12 Glossary

- Transformation
- Translation

### Vertical Alignment

**Previous Benchmarks**
- MA.8.GR.2

**Next Benchmarks**
- MA.912.GR.2

### Purpose and Instructional Strategies

In grade 8, students performed single transformations on two-dimensional figures. In Algebra I, students identify the effects of single transformations on linear, quadratic and absolute value functions. In Geometry, students will perform multiple transformations on two-dimensional figures. In later courses, students will work with transformations of many types of functions.

- In this benchmark, students will examine the impact of transformations on linear, quadratic and absolute value functions. Instruction includes the use of graphing software to ensure adequate time for students to examine multiple transformations on the graphs of functions.
  - Have students use graphing technology to explore different parent functions.
    - In each graph, toggle on/off the graphs for $f(x) + k, kf(x)$, $f(kx)$ and $f(x + k)$ to examine their impacts on the function. Use the slider to change the value of $k$ (be sure to examine the impacts when $k$ is positive and negative).
    - As students explore, prompt discussion (*MTR.4.1*) among them about the patterns they see as they adjust the slider (*MTR.5.1*).

- For $f(x) + k$, students should discover that $k$ is being added to the output of the function (equivalent to the y-value) and will therefore result in a *vertical translation* of the function by $k$ units.

  - Ask students to describe what values of $k$ cause the graph to shift up. Which values cause it to shift down?
- For $kf(x)$, students should discover that $k$ is being multiplied by the output of the function (equivalent to the $y$-value) and will therefore result in a *vertical dilation* (stretch/compression) of the function by a factor of $k$.
  - Ask students to describe what values of $k$ cause the graph to stretch up vertically. Which values cause it to compress? Which values for $k$ cause the graph to reflect over the $x$-axis? What is the significance of $k = -1$?
- For $f(x + k)$, students should discover that $k$ is being added to the input of the function and will therefore result in a *horizontal translation* of the function by $-k$ units.
  - Ask students to describe what values of $k$ cause the graph to shift left. Which values cause it to shift right?
- For $f(kx)$, students should discover that $k$ is being multiplied by the input of the function and will therefore result in a *horizontal dilation* (stretch/compression) of the function by a factor of $k$.
  - Ask students to describe what values of $k$ cause the graph to stretch horizontally. Which values cause it to compress? Which values for $k$ cause the graph to reflect over the $y$-axis? What is the significance of $k = -1$?
- After students have a good understanding of the impact of $f(x) + k$, $kf(x)$, $f(kx)$ and $f(x + k)$ on graphs of functions, connect that knowledge to tables of values for a function.
  - For $f(x) + k$, use graphing technology to display a graph of a quadratic function (like the one below) and set $k = 4$. Guide students to form a table and discuss its connection to the vertical translation observed on the graph.

| $x$ | $f(x)$ | $f(x) + 4$ |
|---|---|---|
| 1 | 6 | 10 |
| 2 | 3 | 7 |
| 3 | 2 | 6 |
| 4 | 3 | 7 |

  - For $kf(x)$, use graphing technology to display a graph of a quadratic function (like the one below) and set $k = 0.5$. Guide students to form a table and discuss its connection to the vertical compression observed on the graph.

| $x$ | $f(x)$ | $0.5[f(x)]$ |
|---|---|---|
| 1 | 6 | 3 |
| 2 | 3 | 1.5 |
| 3 | 2 | 1 |
| 4 | 3 | 1.5 |

  - For $f(x + k)$, use graphing technology to display a graph of a quadratic function (like the one below) and set $k = 2$. Guide students to form a table and discuss its connection to the horizontal translation observed on the graph using the highlighted values. . For the table shown, consider $x = 5$. For $f(x)$, $f(5) = 6$. But for $g(x) = f(x + 2)$, $g(5) = f(5 + 2)$ which is equivalent to 18, which is equivalent to shifting $f(7)$ two units to the left on the graph. Bridge this conversation with a graph of the two functions to help them understand the connection.

| $x$ | $f(x)$ |
|---|---|
| 1 | 6 |
| 2 | 3 |
| 3 | 2 |
| 4 | 3 |
| 5 | 6 |
| 6 | 11 |

→

| $x$ | $g(x) = f(x+2)$ | $g(x)$ |
|---|---|---|
| 3 | 2 | |
| 4 | 3 | |
| 5 | 6 | |
| 6 | 11 | |
| 7 | 18 | |
| 8 | 27 | |

- For $f(kx)$, use graphing technology to display a graph of a quadratic function (like the one below) and set $k = 3$. Guide students to form a table and discuss its connection to the horizontal compression observed on the graph using the highlighted values.

| $x$ | $f(x)$ |
|---|---|
| 1 | 6 |
| 2 | 3 |
| 3 | 2 |
| 4 | 3 |
| 5 | 6 |
| 6 | 11 |
| 7 | 18 |
| 8 | 27 |
| 9 | 38 |

→

| $3x$ | $g(x) = f(3x)$ |
|---|---|
| 3 | 2 |
| 6 | 11 |
| 9 | 38 |
| 12 | 83 |
| 15 | 146 |
| 18 | 258 |
| 21 | 326 |
| 24 | 443 |
| 27 | 578 |

## Common Misconceptions or Errors

- Similar to writing functions in vertex form, students may confuse effect of the sign of $k$ in $f(x+k)$. Direct these students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of $k$.
- Vertical stretch/compression can be hard for students to see on linear functions initially and they may interpret stretch/compression as rotation. Introduce the effects of $kf(x)$ *and* $f(kx)$ by using a quadratic or absolute value function first before analyzing the effect on a linear function.
- Students may think that a vertical and horizontal stretch from $kf(x)$ and $f(kx)$ look the same.

## Strategies to Support Tiered Instruction

- Instruction includes explaining to students that horizontal shifts are "inside" of the function. Additionally, the teacher provides instruction to ensure understanding that the movement of the function is opposite of the sign that effects the horizontal shift.

 - For example, teacher can provide the identification of the type of transformation and its effects to the below function.

$\boldsymbol{f(x-7)+2}$ ← Vertical Shift

Moves 2 units up the y-axis.

↗ Horizontal Shift

Move 7 units to the right on the x-axis.

- Teacher provides instruction that includes the use of a graph that displays stretch and compression (shrink) scaling. Including a visual representation will allow students to categorize their thinking.
  - For example, have students copy the graphs into their notebooks. Give students an opportunity to identify changes in both types of transformations before giving students the transformations.
  - Teachers can also introduce the effects of $kf(x)$ *and* $f(kx)$ by using a quadratic or absolute value function first before analyzing the effect on a linear function.
- Instruction includes providing a grid with a parent function and horizontal and vertical stretch on one grid, using different colors to distinguish both types of stretches (vertical and horizontal).
- Instruction includes directing students to examine a graph of the two functions to see that the horizontal shift is opposite of the sign of $k$.
- Instruction includes having a non-zero $y$-intercept to visualize the difference between scaling in the horizontal direction, $f(kx)$, and scaling in the vertical direction, $kf(x)$.

## Instructional Tasks

*Instructional Task 1 (MTR.3.1)*

Part A. Given the function $f(x) = x^2$, determine the vertex, domain and range.

Part B. If the function $f(x)$ is translated to the right 6 units, predict what may happen to the vertex, domain and range.

Part C. How does the graph of the function $f(x) = x^2 - 7$, compare to the graph of the function in Part A?

## Instructional Items

*Instructional Item 1*

How does the graph of $g(x) = f(x-2)$ compare to the graph of $f(x) = |x+3|$?

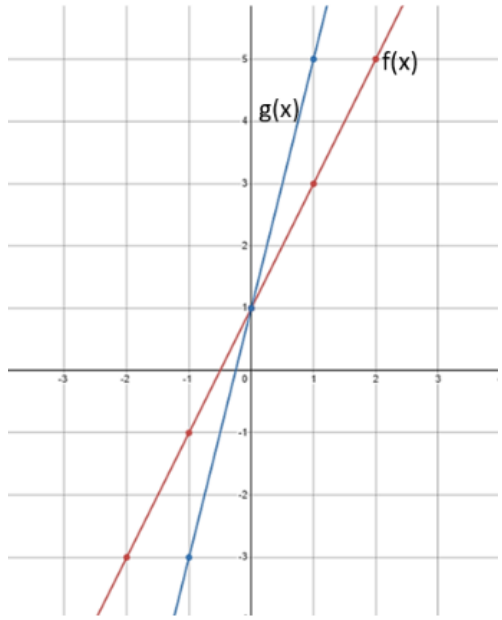
*Instructional Item 2*

Describe the effect of the transformation $f(x) + 2$ on the function table below.

| $x$ | $f(x)$ |
|---|---|
| -2 | 4 |
| 0 | 0 |
| 2 | 4 |
| 4 | 16 |
| 6 | 36 |

*Instructional Item 3*

The graph of two functions is shown below.




Which of the following describes the effect on $f(x)$ after a transformation was performed to produce $g(x)$?

a. The graph of $g(x)$ is reflected over the x-axis of the graph $f(x)$
b. The graph of $g(x)$ is a vertical shift of 2 from the graph $f(x)$
c. The graph of $g(x)$ is a horizontal shift of 1/2 from the graph of $f(x)$
d. The graph of $g(x)$ is a horizontal stretch by a factor of 2 from the graph of $f(x)$

---

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

## Financial Literacy

**MA.912.FL.3** *Describe the advantages and disadvantages of short-term and long-term purchases.*

*MA.912.FL.3.2*

### Benchmark

**MA.912.FL.3.2 Solve real-world problems involving simple, compound and continuously compounded interest.**

*Example:* Find the amount of money on deposit at the end of 5 years if you started with $500 and it was compounded quarterly at 6% interest per year.

*Example:* Joe won $25,000 on a lottery scratch-off ticket. How many years will it take at 6% interest compounded yearly for his money to double*?*

Benchmark Clarifications:
*Clarification 1:* Within the Algebra I course, interest is limited to simple and compound.

### Connecting Benchmarks/Horizontal Alignment

- MA.912.NSO.1.1,
- MA.912.AR1.1, MA.912.AR1.2
- MA.912.AR.2.1, MA.912.AR.2.2, MA.912.AR.2.5
- MA.912.AR.5.3

### Terms from the K-12 Glossary

- Simple Interest

### Vertical Alignment

**Previous Benchmarks**
- MA.7.AR.3.1

**Next Benchmarks**
- MA.912.FL3.3, MA.912.FL.3.4

### Purpose and Instructional Strategies

In grade 7, students solved problems involving simple interest. In Algebra I, students solve problems involving simple and compound interest, using arithmetic operations and graphing. In later courses, students will solve compound interest problems to determine lengths of time, including those that require the use of logarithms, and solve continuously compounded interest problems.

- In this benchmark, students will be introduced to the concepts of simple and compound interest and will solve real-world problems that feature them.
- Compound interest is a method of computing interest. This interest is computed from the (original) principal and the amount of money that has accrued.
- Instruction includes connecting student understanding of MA.912.AR.5.3 using the forms $f(x) = ab^x$ and $f(x) = a(1 \pm r)^x$ to the simple and compound interest formulas. Show students that these formulas are all the same. Explain that b in the standard formula is $(1 \pm r)$.
- Instruction includes connecting student understanding. Instruction compares the differences between simple and compound interest.
  - The Simple Interest Formula ($I = prt$) calculates *only the interest* earned over time.

Each year's interest is calculated from the initial principal, not the total value of the investment of that point in time.
  - The Final Amounts under Simple Interest Formula ($A = P(1 + rt)$) calculates the *total value* of an investment over time. The Final Amounts under Compound Interest formula ($A = P\left(1 + \frac{r}{n}\right)^{nt}$) also calculates the *total value* of an investment over time. Each month/year's interest is calculated from the total value of the investment of that point in time.
- Compound interest problems presented for this benchmark may require students to generate equivalent expressions to identify and interpret certain parts of the context.
  - For example, Jason deposits \$850 in an account that earns an annual interest rate of 4.8%. The interest is compounded monthly, and Jason wants to determine the total amount of interest he will earn in one year. With the given information, derive that the value of the account is equal to $850\ (1 + \frac{0.048}{12})^{12t}$. The expression can be rewritten as $850[(1.004)^{12}]^t$ leading to $850(1.049)t$ to find that the total amount of interest in a year would be approximately 4.9% of his initial investment.

## Common Misconceptions or Errors

- Some problems related to this standard may ask students for the interest earned over a period of time; while others may ask for the account balance or total value of the investment over a period of time. Some students may miss this distinction and may always calculate total interest for simple interest problems and total value for compound interest problems.
- Students may confuse the frequencies of interest being compounded.
- When forming interest equations, students sometimes forget to convert the interest rate from a percent value to a decimal value before substituting it into the formula.

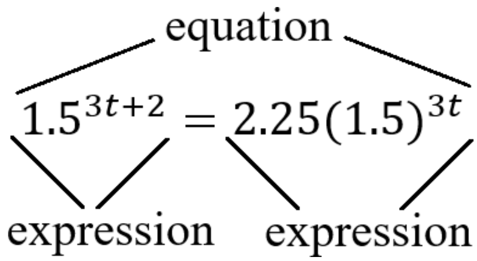
## Strategies to Support Tiered Instruction

- Teacher provides a highlighter to identify if a question is asking for the interest or the total amount. Point students back to the highlighted portions of the problem and help them assess the reasonableness of their answers *(MTR.6.1)* in context.
- Instruction provides a graphic organizer to identify the important information in a problem.
  - For example, given a simple interest problem, students could complete the following table.

| **Principal ($P$)** | **Interest ($I$)** | **Rate ($r$)** | **Time ($t$)** | **Total Value** |
|---|---|---|---|---|
| | | | | |

- Teacher creates an anchor chart to clarify frequencies (annually, quarterly, bi-monthly, etc.) that interest is compounded.
- For students who need extra support in converting a percentage to a decimal, instruction includes students thinking about percent as "per one-hundred."
- For example, when writing 8% as a decimal, ask "8% is how many per 100?" Then write $8\% = \frac{8}{100}$ which is equivalent to 0.08.
- Instruction includes the opportunity to distinguish between an expression and an equation. These should be captured in a math journal.

- For example, when generating equivalent expressions, place an equal sign in between the expressions and label each expression and the equation.




## Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

Felipe signs up for a new airline credit card that has a 24% annual interest rate. If he doesn't pay his monthly statements, interest on his balance would compound *daily*. If Felipe never pays his statements for a full year, what would be the actual percentage rate he would pay the credit card company?

*Instructional Task 2 (MTR.7.1)*

Gwen deposits $800 in a savings account that pays simple annual interest. After 18 months, she earns $64.80. What is the interest rate for her account?

## Instructional Items

*Instruction Item 1*

Beatrice deposits $525 in an account that pays 4.3% simple annual interest. If she keeps the money in the account for 12 years, how much interest will she earn?

*Instructional Item 2*

You deposit $500 in a savings account that pays 2.2% compound annual interest. Find your account balance after 3 years.

---

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.FL.3.4*

## Benchmark

| | |
|---|---|
| **MA.912.FL.3.4** | **Explain the relationship between simple interest and linear growth. Explain the relationship between compound interest and exponential growth and the relationship between continuously compounded interest and exponential growth.** |

Benchmark Clarifications:

*Clarification 1:* Within the Algebra I course; exponential growth is limited to compound interest.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.AR.2.1, MA.912.AR.2.2, MA.912.AR.2.5
- MA.912.F.1.6, MA.912.F.1.8

## Terms from the K-12 Glossary

- Simple Interest

## Vertical Alignment

**Previous Benchmarks**

- MA.7.AR.3.1

**Next Benchmarks**

- MA.912.FL.3.1, MA.912.FL.3.3

## Purpose and Instructional Strategies

In grade 7, students solved problems involving simple interest. In Algebra I, students explain the relationship between simple interest and linear growth and the relationship between compound interest and exponential growth. In later courses, students will extend this to include continuously compounded interest.

- In MA.912.FL.3.2, students became familiar with simple and compound interest and how to use the formulas for each to solve real-world problems. In this benchmark, students will make connections between simple interest and linear growth and between compound interest and exponential growth. To help students discover this relationship, consider guiding them to form a table.
  - For example, Kianna and Samantha both receive \$1,000 cash from graduation gifts from family and friends. They each decide to invest their money in an investment account. Kianna's investment earns 10% in *simple* interest. Samantha's investment earns 10% in *compound* interest annually. Guide students to create the interest formulas below and use them to create the table below to compare the growth of their investments over time.
    - Kianna's Interest Earned would be represented by $A = 1000 \cdot 0.1 \cdot t$.
    - Kianna's Total Value would be represented by $A = 1000(1 + 0.1t)$.
    - Samantha's Interest Earned would be represented by $A = 1000(1 + 0.1)^t - 1000$.
      Samantha's Total Value would be represented by $A = 1000(1 + 0.1)^t$.

| Years Invested | Kianna's Interest Earned (\$) | Total Value of Kianna's Investment (\$) | Samantha's Interest Earned (\$) | Total Value of Samantha's Investment (\$) |
|---|---|---|---|---|
| 1 | 100 | 1,100 | 100 | 1,100 |
| 2 | 200 | 1,200 | 210 | 1,210 |
| 3 | 300 | 1,300 | 331 | 1,331 |
| 4 | 400 | 1,400 | 464.10 | 1,464.10 |
| 5 | 500 | 1,500 | 610.51 | 1,610.51 |
| 10 | 1,000 | 2,000 | 1,593.74 | 2,593.74 |
| 15 | 1,500 | 2,500 | 3,177.25 | 4,177.25 |
| 20 | 2,000 | 3,000 | 5,727.50 | 6,727.50 |
| 30 | 3,000 | 4,000 | 16,449.40 | 17,449.40 |
| 50 | 5,000 | 6,000 | 116,390.90 | 117,390.90 |

  - Once completed, ask students what relationships they observe in the behavior of Kianna's versus Samantha's investment. Students should quickly discover

Kianna's investment exhibits linear growth while Samantha's shows exponential growth.
  - Solidify this understanding by having students graph the two functions that represent the total value of the two investments.
  - Once students make this discovery, begin a conversation with them about which type of interest would be more advantageous for long-term investments. Take this opportunity to make connection to MA.912.F.1.6 (i.e., What if Kianna received $10,000 in gifts? Would the simple interest account be a better investing tool?).
    - Remember the expectation for this benchmark is for students to explain *why* these relationships occur. Be sure to discuss the equations formed and that the variation of years is used as a factor in the simple interest formula and as an exponent in the compound interest formula.

## Common Misconceptions or Errors

- When forming compound interest equations, students sometimes forget to convert the interest rate from a percent value to a decimal value before substituting it into the formula.

## Strategies to Support Tiered Instruction

- Instruction includes making the connection to determining linear and exponential functions (MA.912. F.1.8) from a financial context.
- For students who need extra support in converting a percentage to a decimal, instruction includes students thinking about percent as "per one-hundred."
  - For example, when writing 8% as a decimal, ask "8% is how many per 100?" Then write 8% = 8/100 which is equivalent to 0.08.

## Instructional Tasks

*Instructional Task 1 (MTR.3.1, MTR.4.1, MTR.5.1, MTR.7.1)*

Phoenix invests in a savings account that applies simple interest.

Part A. How will her investment grow, linearly or exponentially? Justify your answer.

Part B. If Phoenix invests $725 and earns an annual rate of 4.2%, write an equation that would represent the total amount she would have at the end of each year.

Part C. How long will it take for her initial investment to double?

Part D. If instead the savings account had interest at the same rate but was compounded annually, how much money would she have after the amount of time found in Part C?)

## Instructional Items

*Instructional Item 1*

Trevarius invests in a savings account that applies compound interest annually. How will his investment grow, linearly or exponentially? Justify your answer.

*Instructional Item 2*

The table shows the balance, in dollars, of a bank account each year after it was opened.

| Year | Balance ($) |
|---|---|
| 0 | 500 |
| 1 | 515 |
| 2 | 530 |
| 3 | 545 |
| 4 | 560 |
| 5 | 575 |

Which of the following describes the interest the account earns and the function that models the balance over time?

a. simple interest; linear function
b. simple interest; exponential function
c. compound interest; linear function
d. compound interest; exponential function

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

## Data Analysis & Probability

**MA. 912.DP.1** *Develop an understanding of statistics and determine measures of center and measures of variability. Summarize statistical distributions graphically and numerically.*

*MA.912.DP.1.1*

### Benchmark

| | |
|---|---|
| **MA. 912.DP.1.1** | **Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate.** |

Benchmark Clarifications:
*Clarification 1:* Instruction includes discussions regarding the strengths and weaknesses of each data display.
*Clarification 2:* Numerical univariate includes histograms, stem-and-leaf plots, box plots and line plots; numerical bivariate includes scatter plots and line graphs; categorical univariate includes bar charts, circle graphs, line plots, frequency tables and relative frequency tables; and categorical bivariate includes segmented bar charts, joint frequency tables and joint relative frequency tables.
*Clarification 3:* Instruction includes the use of appropriate units and labels and, where appropriate, using technology to create data displays.

### Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.3.1

### Terms from the K-12 Glossary

- Bivariate Data
- Categorical Data
- Numerical Data

### Vertical Alignment

**Previous Benchmarks**
- MA.6.DP.1.5
- MA.7.DP.1.5
- MA.8.DP.1.1

**Next Benchmarks**
- MA.912.DP.2.2, MA.912.DP.2.4, MA.912.DP.2.5, MA.912.DP.2.6
- MA.912.DP.3.2, MA.912.DP.3.3
- MA.912.DP.6.6

### Purpose and Instructional Strategies

In middle grades, students used box plots and histograms to display univariate numerical data; then bar charts, circle graphs, line plots and stem and leaf plots to display univariate categorical data; and finally scatter plots and line graphs to display bivariate numerical data. In Algebra I, students display univariate data and bivariate numerical data using graphical representations from middle grades. Students are introduced to bivariate categorical data, which they represent with frequency tables and segmented bar charts. Additionally, they must choose an appropriate display when considering each of the four varieties of data. In later courses, students will build upon this foundation as students consider a variety of data distributions in greater detail, including normal and Poisson distributions.

- While the benchmark states that students select an appropriate data display, instruction also includes cases where students must create the display.
  - This benchmark is closely linked to MA.912.DP.1.2, where students interpret displayed data using key components of the display.
    - Instruction includes student discussions *(MTR.1.1)* regarding the strengths and weaknesses of each data display, and the use of appropriate units and labels *(MTR.4.1)*.
      - Numerical univariate is data that consists of one numerical variable, and an important feature of the data is its numerical size or order. Examples include height, weight, age, salary, speed, number of pets, hours of study, etc. Displays include histograms, stem-and-leaf plots, box plots and line plots.
        - Histograms
          - Good for large sets of data.
          - Shows the shape of the distribution to determine symmetry.
          - Data is collected in suitably sized numerical bins with equal ranges.
          - Because of the bins, only approximate values of individual data points are displayed.
        - Stem-and-Leaf Plots
          - Good for small data sets.
          - Shows the shape of a data set and each individual data value.
          - Lists exact data values in a compact form.
        - Box Plots
          - Beneficial when large amounts of data are involved or compared. Used for descriptive data analysis.
          - Shows multiple measures of variation and/or spread of data.
          - Shows one measure of central tendency (median).
          - Individual data points are not shown.
          - Presents a 5-number summary of the data.
          - Can indicate if a data set is skewed or not, but not the overall shape.
          - Can be used to determine if potential outliers exist.
        - Line Plots (Dot Plots)
          - Used for small to moderate sized data sets in which the numerical values are discrete (often integers, or multiples of ½).
          - Shows the shape of the distribution and the individual data points.
          - Useful for highlighting clusters, gaps and outliers.
          - Can be used to determine measures of center and variation.
      - Numerical bivariate is data that involves two different numerical variables that have a possible relationship to each other. Displays include scatter plots and line graphs.
        - Scatter Plots

        - Good for large data sets, and for data sets in which it is not clear which variable, if any, should be considered the independent variable.
        - Good for showing trends.
      - Line Graphs
        - Good for showing trends or cyclical patterns in small or medium-sized data sets in which there is an independent variable and a dependent variable. Often the values of the independent variable are chosen in advance by the person gathering the data. Examples of independent variables may be points in time or treatment amounts and examples of dependent variables might be total sales or average growth.
    - Categorical univariate is non-numerical data of only one variable that can be categorized/grouped. Displays include bar charts, line plots, circle graphs, frequency tables and relative frequency tables.
      - Bar Charts (Bar Graphs)
        - Good for showing comparisons between categories or between different populations.
        - A bar chart may show frequencies (counts) or relative frequencies (percentages) in each category.
      - Circle Graphs
        - Good for illustrating the percentage breakdown of items and visually representing a comparison.
        - Not effective when there are too many categories.
        - Shows how categories represent parts of a whole.
        - A circle graph may show frequencies (counts) or relative frequencies (percentages) in each category.
      - Frequency Tables and Relative Frequency Tables
        - This is often the easiest way to display bivariate categorical data. The categories for one variable are listed in the header row of the table and the categories for the other variable are listed in the header column.
        - The frequencies (counts) or relative frequencies (percentages) are listed in the cells for each of the indicated joint categories. Total counts or percentages for the rows may be listed in the final column of the table and total counts or percentages for the columns may be listed in the final row.
      - Segmented Bar Charts
        - Comparison of more than one categorical data set.
- Good for showing the composition of the individual parts to the whole and making comparisons. Non-numerical data may consist of numbers if the categories are not primarily determined by the numerical size or order of the numbers.
    - For example, the data may answer the question “What is your favorite real number?” and the categories could be “Integers,” “Rational numbers that are not integers” and “Irrational numbers.”
- Using the same real-world data *(MTR.7.1)*, encourage students to create a variety of

data displays appropriate for the data given *(MTR.2.1)*. This makes the discussion of the similarities and differences of the displays more robust and allows students to visualize and justify their responses *(MTR.3.1)*.
  - This strategy might work best if you present the class with a set of data, group students and ask each group to create a different display using the same data.
  - Each group can then present the strengths and weaknesses of their display as compared to the others *(MTR.5.1)*.
  - This should be repeated for each separate data category. See examples above.
- This benchmark references bar charts; however, other benchmarks and the glossary (Appendix C) reference bar graph, these terms are used interchangeably without difference.

## Common Misconceptions or Errors

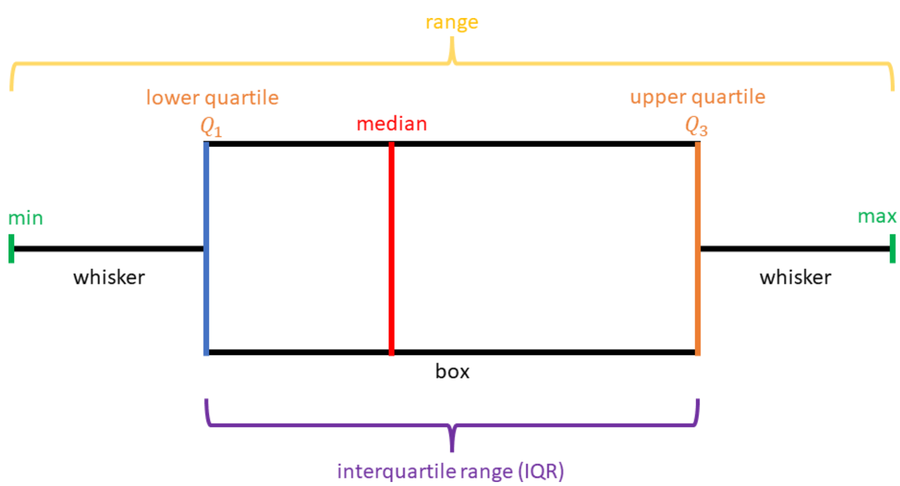
- Students may not know how to label displays appropriately or how to choose appropriate units and scaling.
  - For example, they may not know how to create or scale the number line for a line plot, they may confuse frequency and actual data values, or they may not understand that intervals for histograms should be done in equal increments.
- Students may not understand the meaning of quartiles in the box plot.
- Students may not know how to calculate the median with an even number of data values.
- Students may not accurately place data values in increasing order when there are many data points.
- Students may confuse bar charts (for categorical data) and histograms (for numerical data).
- Students may be confused when categorical data consists of numbers that have been categorized in ways that do not primarily reflect the numerical size or order of the numbers. In such cases, it will be helpful to have the student think about whether any of the measures of center (mean, median) or variability (quartiles, range) are meaningful for the data set. If they are, then the data can be considered numerical, because these measures are concerned with the numerical size and order of the data points. If not, then it can be considered categorical.

## Strategies to Support Tiered Instruction

- Teacher co-creates anchor charts that include appropriate units of measure.
  - For example, time measurement units include seconds, minutes, hours, days, weeks, etc.
- Teacher provides numerical univariate, numerical bivariate, categorical univariate and categorical bivariate data examples. Each example should include scaling to ensure that students have experience scaling for graphs and tables that are in each category.
  - For example, employee ages for the company AdvertiseHere can be displayed using a box plot as shown.


- Teacher reviews the difference between histograms and bar graphs, creating an anchor chart with properties of a histogram for students to refer to.
- Teacher reinforces how scales are represented with specific endpoints. The endpoints they chose to use, or as defined in a problem, tell them if the point is included in the bin or not. Include notation of endpoints on anchor chart to display in the classroom.
- Teacher co-constructs vocabulary guide/anchor chart with students who need additional support understanding the vocabulary for measures of center and variation.
    - Examples of guides and charts are shown below.

| Term | Definition | How it is found or calculated |
|---|---|---|
| Mean | | |
| Median | | |
| Mode | | |
| Range | | |
| Interquartile Range (IQR) | | |
| Quartiles | | |




- Teacher models ordering data sets in ascending order before finding a median, quartile or range.
- Teacher provides a chart to display calculating the median with an even and odd data set.

| Odd Number of Data | Even Number of Data |
|---|---|
| The Middle Number | Average of the Two Middle Numbers |
| $\{1, 2, 2, 2, 3, 4, 5\}$<br><br>The median is 2. | $\{1, 2, 3, 4, 5, 6, 7, 8\}$<br><br>The median is the average (mean) of 4 and 5 which is 4.5. |

- Instruction includes discussions about whether any of the measures of center (mean, median) or variability (quartiles, range) are meaningful for the data set. If they are, then the data can be considered numerical, because these measures are concerned with the numerical size and order of the data points. If not, then it can be considered categorical.

## Instructional Tasks

*Instructional Task 1 (MTR.7.1)*

The following data set shows the change in the total amount of municipal waste generated in the United States during the 1990's.

| Year | Municipal Waste Generated (Millions of Tons) |
|---|---|
| 1990 | 269 |
| 1991 | 294 |
| 1992 | 281 |
| 1993 | 292 |
| 1994 | 307 |
| 1995 | 323 |
| 1996 | 327 |
| 1997 | 327 |

Figure: Total Municipal Waste Generated in the US by Year in Millions of Tons.

*Instructional Task 2 (MTR.3.1, MTR.7.1)*

High school students in the United States were invited to complete an online survey in 2010. More than 1,000 students responded to this survey which included a question about a student's favorite sport. 450 of the completed surveys were randomly selected. A breakdown of the data by gender was compiled from the 450 surveys.

- 100 students indicated their favorite sport was soccer. 49 of those students were females.
- 131 students selected lacrosse as their favorite sport. 71 of those students were males.
- 75 students selected basketball their favorite sport. 48 of those students were females.
- 26 students indicated football as their favorite sport. 25 of those students were males.
- 118 students indicated volleyball as their favorite sport. 70 of those students were females.

Choose and create an appropriate data display to represent the information given.

## Instructional Items

*Instructional Item 1*

The following table shows the total weight in tons of the most common types of electronic equipment discarded in the United States in 2005.

| **Electronic Equipment** | **Thousands of Tons Discarded** |
|---|---|
| Cathode Ray Tube (CRT) TV's | 7591.1 |
| CRT Monitors | 389.8 |
| Printers, Keyboards, Mice | 324.9 |
| Desktop Computers | 259.5 |
| Laptop Computers | 30.8 |
| Projection TV's | 132.8 |
| Cell Phones | 11.7 |
| LCD Monitors | 4.9 |

Figure: Electronics Discarded in the US (2005). Source: National Geographic, January 2008. Volume 213 No.1, pg 73.

Which data display would you use to represent this data? Explain your reasoning.

*Instructional Item 2*

The number of cars sold in a week at a large car dealership over a 20-week period is given below.

16 12 8 7 26 32 15 51 29 45 19 11 6 15 32 18 43 31 23 23

Which data display would be an inappropriate way to represent this data? Explain your reasoning.

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.DP.1.2*

## Benchmark

| | |
|---|---|
| **MA.912.DP.1.2** | **Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display.** |

Benchmark Clarifications:

*Clarification 1*: Within the Probability and Statistics course, instruction includes the use of spreadsheets and technology.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.3.1

## Terms from the K-12 Glossary

- Bivariate Data
- Categorical Data
- Numerical Data

## Vertical Alignment

**Previous Benchmarks**
- MA.6.DP.1
- MA.7.DP.1
- MA.8.DP.1

**Next Benchmarks**
- MA.912.FL.4.4
- MA.912.DP.2.1, MA.912.DP.2.2, MA.912.DP.2.4, MA.912.DP.2.5, MA.912.DP.2.6
- MA.912.DP.3.2, MA.912.DP.3.3

## Purpose and Instructional Strategies

In grade 7, students created and interpreted different displays of univariate numerical and categorical data. They also used measures of center and variation to make comparisons and draw conclusions about two populations. In grade 8, students created scatter plots and began to interpret them by considering lines of fit. In Algebra I, students interpret the components of data displays for numerical and categorical data, both univariate and bivariate. In later courses, they will use data displays to compare distributions of data sets to one another and to theoretical distributions.

- It is the intention of this benchmark to include cases where students must calculate measures of center/variation to interpret *(MTR.3.1)*.
- For students to have full understanding of numerical/categorical, univariate/bivariate data sets and their displays, instruction should include MA.912.DP.1.1. These benchmarks are not intended to be separated. One is reinforced by the other.
  - Numerical univariate includes histograms, stem-and-leaf plots, box plots and line plots.
  - Numerical bivariate includes scatter plots and line graphs.
  - Categorical univariate includes bar charts, line plots, circle graphs, frequency tables and relative frequency tables.
  - Categorical bivariate includes segmented bar charts, joint frequency tables and joint relative frequency tables.
- Instruction includes identifying the measures of center and spread from different scenarios.
- Instruction includes reviewing that an outlier is significantly smaller or larger than the rest of the data set. A data set may have more than one outlier.
- Teacher provides opportunities to analyze the effects on the measures of center and spread when the outlier is the minimum or maximum.
- This benchmark reinforces the importance of the use of questioning within instruction.
  - Does this display univariate or bivariate data?
  - Is the data numerical or categorical?
  - What do the different quantities within the data display mean in terms of the context of the situational data?

## Common Misconceptions or Errors

- Students may not be able to properly distinguish between numerical and categorical data or between univariate and bivariate data.
- Students may misidentify or misinterpret the quantities in data displays.
- Students may not be able to distinguish between the measures of center (mean, median) and measures of spread (range, IQR).
- Students may not completely grasp the effect of outliers on the data set or incorrectly conclude a point is an outlier.
- Students may not be able to distinguish the differences between frequencies and relative frequencies.
- Students misidentify the condition that determines a conditional or relative frequency in a joint table.

## Strategies to Support Tiered Instruction

- Instruction includes a graphic organizer to complete collaboratively.
  - For example, teacher can provide the graphic below and have students match the vocabulary terminology with the correct definition. Then, have students create an example that can help with remembering the vocabulary terminology.

| **Word Bank** | |
|---|---|
| Numerical | Univariate |
| Categorical | Bivariate |
| **Fill in the sentence.** | **Example** |
| [______] data is measured with real numbers. | |
| [______] data is described using two numbers or two categories. | |
| [______] data is separated into groups . | |
| [______] data is described using one number or category. | |

- Teacher provides students with definitions and co-creates examples for frequency and relative frequency.
  - For example, have students draw a definition chart in their notebook. Give them the opportunity to create an example that will help them remember the definition.

| **Term** | **Frequency** | **Relative Frequency** |
|---|---|---|
| **Definition** | The number of data points in each category. | The number of data points in a category divided by the overall total. |
| **Example** | | |

## Instructional Tasks

*Instructional Task 1 (MTR.3.1, MTR.4.1)*

The histogram below shows the efficiency rate (in miles per gallon) of 110 cars.

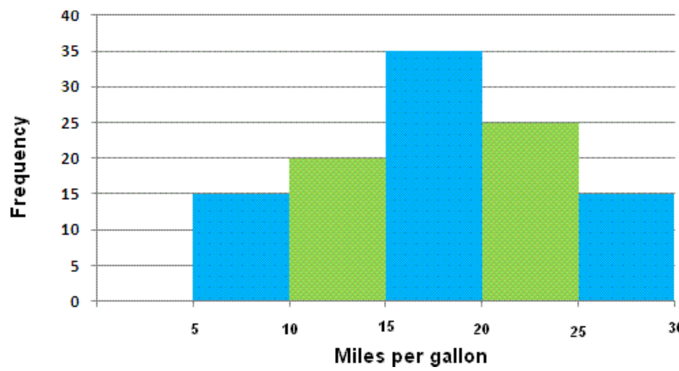
Part A. Does this display univariate or bivariate data?

Part B. Is the data numerical or categorical?

Part C. What do the different quantities within the data display mean in terms of the context of the situational data?

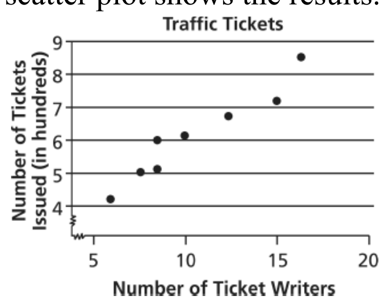
Part D.

- How many cars have an efficiency rate between 15 and 20 miles per gallon?
- How many cars have an efficiency rate more than 20 miles per gallon?
- What percentage of cars have an efficiency rate less than 20 miles per gallon?




*Instructional Task 2 (MTR.3.1, MTR.4.1)*

A police department tracked the number of ticket writers and number of tickets issued for each of the past 8 weeks. The scatter plot shows the results.




Part A. Does this display univariate or bivariate data? Is the data numerical or categorical?

Part B. What do the different quantities within the data display mean in terms of the context of the situational data?

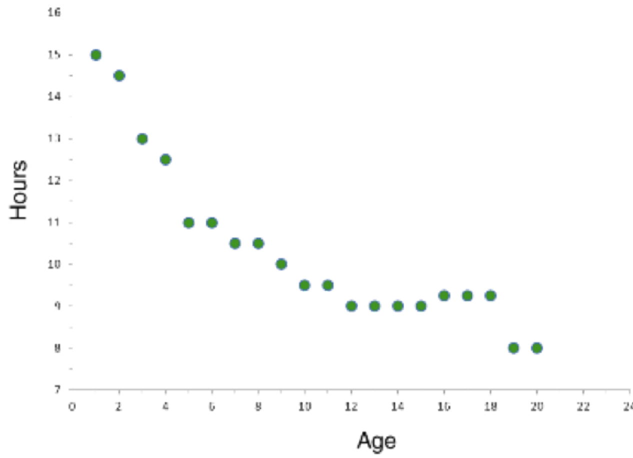
Part C. Which statement is an appropriate interpretation of the data?

a. More ticket writers result in fewer tickets being issued.
b. There were 50 tickets issued every week.
c. When there are 10 ticket writers, there will be 600 tickets issued.
d. When there are more ticket writers, more tickets are being issued.

## Instructional Items

*Instructional Item 1*

The scatter plot shows the amount of sleep needed per day by age.

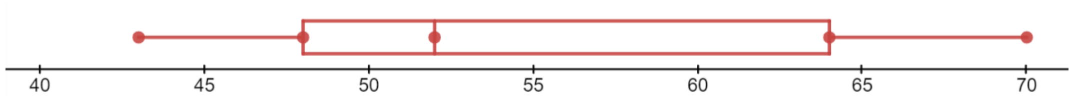



Part A. Does this display univariate or bivariate data? Is the data numerical or categorical?

Part B. What is a possible trend that is shown by the data?

*Instructional Item 2*

The box plot shows the scores of students for the most recent quiz.




Part A. Does this display univariate or bivariate data? Is the data numerical or categorical?

Part B. What is the maximum, minimum, lower quartile, upper quartile and median?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

*MA.912.DP.1.4*

## Benchmark

**MA.912.DP.1.4 Estimate a population total, mean or percentage using data from a sample survey; develop a margin of error through the use of simulation.**

*Algebra I Example:* Based on a survey of 100 households in Twin Lakes, the newspaper reports that the average number of televisions per household is 3.5 with a margin of error of ±0.6. The actual population mean can be estimated to be between 2.9 and 4.1 television per household. Since there are 5,500 households in Twin Lakes the estimated number of televisions is between 15,950 and 22,550.

Benchmark Clarifications:
*Clarification 1:* Within the Algebra I course, the margin of error will be given.

## Connecting Benchmarks/Horizontal Alignment

## Terms from the K-12 Glossary

- Random Sampling
- Simulation

## Vertical Alignment

**Previous Benchmarks**

- MA.7.AR.3
- MA.7.DP.1
- MA.7.DP.2
- MA.8.DP.2

**Next Benchmarks**

- MA.912.DP.1.5
- MA.912.DP.2.3, MA.912.DP.2.4, MA.912.DP.2.6

## Purpose and Instructional Strategies

In grade 7, students solved real-world problems involving percentages and they calculated means of numerical data sets. In grades 7 and 8, students explored the relationship between theoretical probabilities and experimental probabilities. In Algebra I, students use means and percentages from data obtained from statistical experiments to make predictions about actual means and percentages in populations and they relate the experimental data to actual values through margins of error. In later courses, students will work more formally with margins of error and use the normal distribution to estimate population percentages.

- In Algebra I, students are not expected to master the skill of estimating a margin of error using simulation. But instruction may include such an activity, to allow students to experience the need for a margin of error whenever a mean or population percentage is estimated using data.
- Instruction includes introducing examples from the media of reports of population means and percentages that include margins of error.

- Instruction includes the understanding that an actual population mean or percentage is not necessarily within the margin of error of the estimated mean or percentage. Even if the data has been carefully collected, these estimated quantities are only within the margin of error with a “high degree of confidence.” If the data has not been carefully collected, or if the situation has changed significantly since the data were collected (as is often the case with election polls), the margin of error may not be very meaningful.

## Common Misconceptions or Errors

- When asked for population totals, students may forget to complete the final calculation with the margin of error and only report the mean or percentage as their final answer.
- Students may confuse three percentages: the estimated population percentage, the margin of error expressed as a percentage and the actual population percentage. Students can use a number line to visualize how the first two quantities determine an interval that is likely to contain the third quantity.

## Strategies to Support Tiered Instruction

- Teacher provides a list of definitions to students to eliminate misunderstandings that may be caused by the key terms. Because it is important that students understand the meanings of the terms when interpreting sample surveys, this should be an entry in a math journal.
- Instruction includes providing multiple scenarios and asking for identification of the key terms associated with the percentages given in the context.
- Teacher models the use of a number line to visualize how the estimated population percentage and the margin of error determine an interval that is likely to contain the actual population percentage.

## Instructional Tasks

*Instructional Task 1 (MTR.4.1)*

Based on a survey of 150 students at Long Lake High School, the average number of hours spent on social media per week is 30.5 with a margin of error of 3.25.

Part A. Give a range of values, based on this data, for the actual mean numbers of hours spent on social media.

Part B. If there are a total of 723 students at the high school, give a range of values for the total number of weekly hours spent by all students.

## Instructional Items

*Instructional Item 1*

A packaging company prints 100,000 boxes per day. They determine that their printing machines are making a mistake on an average of 0.11% of boxes per day with a margin of error of $\pm 0.01\%$. How many boxes will need to be recycled due to a printing error in one week?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*

**MA.912.DP.3** *Solve problems involving categorical data.*

*MA.912.DP.3.1*

## Benchmark

**MA.912.DP.3.1** **Construct a two-way frequency table summarizing bivariate categorical data. Interpret joint and marginal frequencies and determine possible associations in terms of a real-world context.**

*Algebra I Example:* Complete the frequency table below.

| | Has an A in math | Doesn't have an A in math | Total |
|---|---|---|---|
| Plays an instrument | 20 | | 90 |
| Doesn't play an instrument | 20 | | |
| Total | | | 350 |

Using the information in the table, it is possible to determine that the second column contains the numbers 70 and 240. This means that there are 70 students who play an instrument but do not have an A in math and the total number of students who play an instrument is 90. The ratio of the joint frequencies in the first column is 1 to 1 and the ratio in the second column is 7 to 24, indicating a strong positive association between playing an instrument and getting an A in math.

## Connecting Benchmarks/Horizontal Alignment

- MA.912.DP.1.1, MA.912.DP.1.2

## Terms from the K-12 Glossary

- Bivariate Data
- Categorical Data
- Joint Frequency

## Vertical Alignment

**Previous Benchmarks**

- MA.7.DP.1
- MA.8.DP.1

**Next Benchmarks**

- MA.912.DP.2.4, MA.912.DP.2.6
- MA.912.DP.4.5, MA.912.DP.4.6

## Purpose and Instructional Strategies

In grades 7 and 8 students explored the relationship between experimental and theoretical probabilities. In Algebra I, students study bivariate categorical data and display it in tables showing joint frequencies and marginal frequencies. In later courses, students will study experimental and theoretical conditional probabilities.

- Instruction includes the connection to MA.912.DP.1.1 where students work with categorical bivariate data and display it in tables. A two-way frequency table is a way to display frequencies jointly for two categories.

- In order to interpret the joint and marginal frequencies, students must know the difference between the two.
  - Marginal frequencies
    Guide students to understand that the total column and total row are in the "margins" of the table, thus they are referred to as the marginal frequencies.
  - Joint frequencies
    Guide students to connect that the word joint refers to the coming together of more than one, therefore the term joint frequency refers to combination of two categories or conditions happening together.
- Once the two-way table is complete, students can compare two ratios to assess the association of the data.
  - Comparing Joint Frequencies
    They can either compare the ratios of the two joint frequencies of each of the columns (as was done in the Benchmark Example) or they can compare the ratios of the two joint frequencies in each of the rows.
    - For example, a completed two-way frequency table is shown below and one wants to determine the association between owning a dog and being male. The ratio of the joint frequencies in the first row is $\frac{14}{32}$, which is about 0.45, and the ratio of the joint frequencies in the second row is $\frac{18}{37}$, which is about 0.49. Since 0.45 and 0.49 are very close to one another, one can conclude that there is no association between owning a dog and being male. But since the ratio of the first row is slightly smaller than the ratio in the second row, one could argue that there is a weak, negative association. Likewise, since the ratio of the second row is slightly larger than the ratio in the first row, one could conclude that there is a weak, positive association between owning a dog and being female.

| | Owns a dog | Does not own a dog | Total |
|---|---|---|---|
| Male | 14 | 31 | 45 |
| Female | 18 | 37 | 55 |
| Total | 32 | 68 | 100 |

- Comparing Joint and Marginal Frequencies
  They can either compare the ratios of a joint and a marginal frequency of each of the columns or they can compare the ratios of a joint and a marginal frequency in each of the rows.
  - For example, a completed two-way frequency table is shown below and one wants to determine the association between owning a dog and being male. The ratio of the joint frequency of males and the marginal frequency in the first column is $\frac{14}{32}$, which is about 0.44, and the ratio of the joint frequency of males and the marginal frequency in the second column is $\frac{31}{68}$, which is about 0.46. Since 0.44 and 0.46 are very close to one another, one can conclude that there is no association between owning a dog and being male. But since the ratio of the first column is slightly smaller than the ratio in the second column, one could argue that there is a weak, negative association. Alternately, one can compare the ratio of the joint frequency of females and the marginal frequency in the first column is $\frac{18}{32}$, which is about 0.56, and the ratio of the joint frequency of females and the marginal frequency in the second column is $\frac{37}{68}$, which is about 0.54. Since the ratio of the first column is slightly larger than the ratio of the second column, one could conclude that there is a weak, positive association between owning a dog and being female.

| | Owns a dog | Does not own a dog | Total |
|---|---|---|---|
| Male | 14 | 31 | 45 |
| Female | 18 | 37 | 55 |
| Total | 32 | 68 | 100 |

- Instruction focuses on frequency tables where each of the variables only has two categories (i.e., likes ice cream and does not like ice cream).

## Common Misconceptions or Errors

- Students may not be able to properly complete the table based on the data given.
- Students may not be able to distinguish the differences between marginal and joint frequencies.
- Students may not be able to properly identify the relationships and possible associations in the data in terms of the context given.
- When determining association, students may not be able to assess whether the association is positive or negative since it is based on how you state your justification.
  - For example, in the study above about gender and owning a dog, the association between owning a dog and being male is negative which is equivalent to saying that the association between owning a dog and being female is positive.

## Strategies to Support Tiered Instruction

- Instruction includes writing the definitions of joint frequency and marginal frequency vocabulary terms in interactive journals.
  - For example, students can list the definitions (shown below) inside of their notebooks.

| Joint Frequency | Marginal Frequency |
|---|---|
| Category values inside of the body of a table. | Totals found in both the rows and columns in a table. |

Joint Frequency:

| | 6th Grade | 7th Grade | 8th Grade | Total |
|---|---|---|---|---|
| Cat | 7 | 3 | 10 | 20 |
| Dog | 11 | 15 | 9 | 35 |
| Other | 4 | 14 | 2 | 20 |
| Total | 22 | 32 | 21 | 75 |

Marginal Frequency:

| | 6th Grade | 7th Grade | 8th Grade | Total |
|---|---|---|---|---|
| Cat | 7 | 3 | 10 | 20 |
| Dog | 11 | 15 | 9 | 35 |
| Other | 4 | 14 | 2 | 20 |
| Total | 22 | 32 | 21 | 75 |

- Instruction includes the use of a key word table associated with joint and marginal frequency. Instruction also includes the use of a reading strategy that gives students the opportunity to analyze the data from the context given.
- Teacher provides additional assistance when identifying positive and negative associations. Instruction includes using categories that are relevant to students to increase interest and comfortability.
  - For example, the following context could be relevant to students. Students at a local junior high school were surveyed. The survey wanted to determine if the group of students are in an athletic club or a social club. Can you draw any associations from the given data?

| | | **In an Athletic Club** | | |
|---|---|---|---|---|
| | | **Yes** | **No** | **Total** |
| **In a Social Club** | **Yes** | 72 | 37 | **109** |
| | **No** | 41 | 29 | **70** |
| | **Total** | **113** | **66** | **179** |

## Instructional Tasks

*Instructional Task 1 (MTR.5.1)*

A survey asked, "If you could have a new vehicle, would you want a sport utility vehicle or a sports car?" Below are the results of the survey in a joint relative frequency table.

| | Sport Utility Vehicle (SUV) | Sports Car | Total |
|---|---|---|---|
| Male | | 42 | 67 |
| Female | 145 | | |
| Total | | 97 | |

Part A. Construct a frequency table.
Part B. Does this display univariate or bivariate data? Is the data numerical or categorical?
Part C. What percentage of the survey takers was female?
Part D. Did a higher percentage of the males or females choose an SUV?
Part E. Does the data show an association between being female and choosing an SUV? If so, is it positive or negative?

## Instructional Items

*Instructional Item 1*

Complete the frequency table below based on data collected randomly from 1000 people at the local county fair.

| | Owns a Motorcycle | Do not Own a Motorcycle | Total |
|---|---|---|---|
| Men | 506 | | |
| Women | 72 | | 375 |
| Total | | | |

Is there an association between being a man and owning a motorcycle?

**The strategies, tasks and items included in the B1G-M are examples and should not be considered comprehensive.*