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The NCTM *Standards* in a Japanese Primary School Classroom

Valuing Students' Diverse Ideas and Learning Paths

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CREATING a learning environment where individual students' ideas, interests, and experiences are valued is important. It can help inform teachers where their students are and how to guide the class so that their students develop a good understanding of the content while expressing their own thinking. However, creating such an environment is not a simple matter. In the classroom context, where teachers are expected to meet the needs of students of diverse backgrounds while adapting themselves to the changing contexts of teaching, the complexity of the challenge can be overwhelming.

The result of the Third International Mathematics and Science Study (TIMSS) indicated that Japanese elementary school mathematics classrooms reflected many characteristics of U.S. reforms designed to develop such learning environments. The open-ended problem solving in Japanese classrooms and discussion-centered approaches effectively draw students' ideas out into the open where teachers can facilitate students' understanding (TIMSS 1997). Several comparative studies have also demonstrated that

The research reported in this article was supported in part by the National Science Foundation (NSF) under grant no. REC-9806020 and in part by the Spencer Foundation. The opinions expressed in this paper are those of the authors and do not necessarily reflect the views of NSF or the Spencer Foundation.

Japanese classrooms are qualitatively different from U.S. classrooms in both teachers' expectations of what learning is and students' academic performance (e.g., Inagaki, Morita, and Hatano 1999; Stigler and Hiebert 1999). Japanese teachers are guided by the beliefs that all students will be able to do mathematics with enough time and practice and that individual developmental differences must be recognized. Consequently, their questions are sequenced to become gradually more complex as students gain experiences that support their learning. A safe classroom environment helps students share their ideas, make mistakes, and learn together. This qualitatively different classroom image closely reflects many ideas presented in the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (NCTM 2000).

In many U.S. classrooms where teachers are serious about incorporating reform ideas in their teaching, we see images of similarly effective teaching practices (e.g., Fuson et al. 2000; Lampert and Ball 1998). This article extends our existing knowledge of effective teaching practice by examining aspects of Japanese teaching that are closely aligned with the Teaching Principle (NCTM 2000). Seeing the U.S. reform ideas in action in a Japanese classroom enables U.S. teachers to see how the elements of the Teaching Principle are practiced in a different cultural context, how Japanese teachers help students learn, and how certain aspects of Japanese practice might be incorporated in instruction in the United States.

We discuss (*a*) how valuing students' diverse ideas and learning paths helps create an effective mathematics-learning environment and (*b*) the roles of different visual learning supports in such an environment. Although we are using an example from grade 1, the ideas are applicable throughout the elementary school grades.

SCHOOL AND PARTICIPANTS

Green Leaf Community School is a full-day Japanese school in a suburb of Chicago. It is one of the two full-day schools in the United States that are operated by the Japanese Ministry of Education and that closely follow the Japanese National Course of Study. Administrators and teachers are sent directly from Japan through the ministry, and the instructional language is Japanese. The second and third authors of this article are Japanese teachers who taught at the school. The school houses approximately 200 students in first through ninth grades. These students typically come to the United States because of their fathers' work in the area, and the families who relocate under these circumstances typically stay in the United States for two to five years and then return to Japan. They are different from other ethnic minority groups in the United States who immigrate to stay. This Japanese community puts much effort into preserving the Japanese culture in their children's lives because they wish for their children to be successful on returning to Japan. For that reason, maintaining Japanese ways of teaching and learning is considered to be important.

This paper summarizes the intensive data collected in one grade 1 classroom of twenty-five students as they engaged in learning teens additions. This is one of the core units of grade 1, and it was typical of the teaching observed throughout the year and in other grade levels. Learning the break-apart-to-make-ten method is viewed as crucial for later multidigit calculation because it enables students to think about teen numbers as "10 + another number" to make regrouping ("carrying") more sensible. The unit was taught during the fifth month of the school year and consisted of eleven lessons over a three-week period. Since the beginning of the school year, grade 1 students had explored numbers and counting up to 10, decomposition and recomposition of numbers smaller than 10, addition and subtraction of numbers with 10 to make teens without regrouping (e.g., $10 + 3 = 13$, $18 - 8 = 10$), and addition and subtraction with three numbers (e.g., $8 + 2 + 4 = 14$, $17 - 7 - 2 = 8$).

Learning to group numbers in tens may be considered important mathematically across cultures because of the worldwide base-ten system. However, in Japan, making tens is also culturally important, and Japanese students have opportunities outside the classroom to experience the "ten-ness" in their everyday lives. The Japanese language, like other Chinese-based languages, explicitly states "ten" in teen numbers (e.g., 11, 12, 13 are said as "ten-one," "ten-two," and "ten-three") and in the decade numbers (e.g., 50 is said as "five ten"), and thus students hear and experience ten within numbers. Japanese students may feel the "bumps" as they hear the numbers "ten-one," "ten-two," and "ten-three," and this gives them a clue that after ten, a new sequence starts as the counting in the ones column starts over (Easley 1983; Fuson and Kwon 1992).

Students also see things typically grouped in tens to be sold in stores in Japan, such as eggs in a carton of ten (instead of a dozen) and bottled drinks are packaged as a ten-pack instead of a six-pack. Smaller packages of five items are also available, such as postcards, and pairs of socks may be sold in packages of five. Because of their use of the metric system, too, students hear and experience how groupings of ten are used for measuring their everyday items. Such experiences contribute to young Japanese students' development of a ten-structured understanding of quantities. The learning of the break-apart-to-make-ten method in grade 1 is considered an extension of students' experiences of ten-ness in their everyday lives—experiences that support the development of their future place-value understanding. Japanese students' experiences are more uniformly consistent than those of their U.S. peers, who use the base-ten system in classrooms but who have their everyday experiences with quantities that depend on different reference numbers (e.g., 12 for dozen, 16 for weight conversion between pounds and ounces).

Although the classroom was observed during the teens addition unit, careful field notes were taken to record (1) the general lesson procedures, (2) teaching and learning activities and their structures, (3) the kinds of student participation in the activities, (4) students' engagement and reaction to the activities, (5) verbatim transcripts of whole-class and small-group discussions (as many as possible of the latter), (6) questions and responses of teacher and students, and (7) conversations among researcher, teacher, and students before, during, and after the class. The lessons were also videotaped to support and supplement the observation notes. Classroom artifacts (copies of worksheets, tests, quizzes, etc.) were also collected.

VALUING STUDENTS' IDEAS

As the lessons unfolded in the teens addition unit, the teacher carefully orchestrated discussions so that students shared their different ideas that were based on their prior understanding of the decomposition of numbers (e.g., 4 is decomposed into 1 and 3). The major parts of the first two lessons were devoted to students' sharing various methods to understand the addition situation of "9 + 4" by decomposing two addends into different quantities that were easier to add. The unit followed the National Course of Study and was designed to introduce grade 1 students to "9 + another number" problems first and then proceed to "8 + another number," then to "7 + another number" problems. This helped students to extend their prior understanding of the decomposition of 10 by focusing on certain break-apart pairs (e.g., 9 and 1, 8 and 2). Instead of focusing on the same total number with different addends to make that total (e.g., 13 for 9 + 4, 8 + 5, 7 + 6), the Japanese approach of grouping problems with the first addend helps students do the first step of the break-apart-to-make-ten method.

For the initial introduction, the teacher showed a group of nine blue magnetic counters and a group of four red magnetic counters on the chalkboard (fig. 7.1). Some students immediately shouted out the answer, "13!" The teacher then initiated discussion by saying, "Some of you are quick in telling the answer, but who can share with the class your thinking?"

Several students raised their hands to share their ideas. Sakiko went up to the board when called on and moved one red counter to add to the blue group.

Sakiko: From 4, I add 1 to 9 and make 10.

Teacher: So, the 9 became 10 and the 4 became 3?

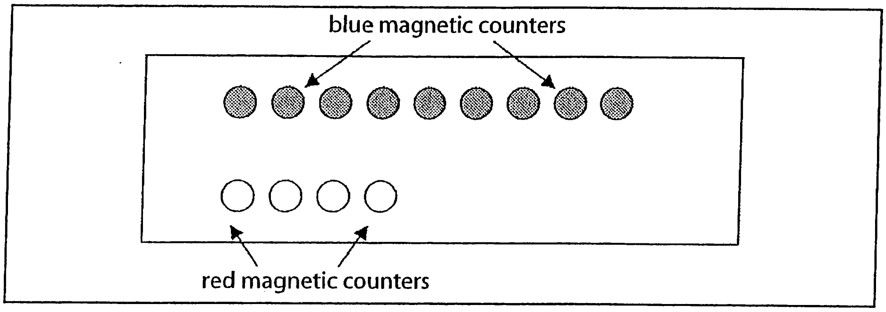

Fig. 7.1. Magnetic counters on the board to guide students' thinking for 9+4

Sakiko: Then we know 10 and 3 make 13. We learned that before [*in a previous unit*].

Tadashi then volunteered to show the class how he counted unitarily, 1 to 13, to get the answer. Following his contribution, several students shared different counting methods: Tetsuhiro counted by twos, and Tomoko counted by threes. Although counting was not the official topic of the lesson, the teacher allowed students time to explain, and he then summarized to the whole class how counting by ones, twos, and threes were different from one another.

Nobuhiko raised his hand. He came to the board to show how he solved the problem, 9 + 4.

Nobuhiko: I made 9 into 7 and 2, and then thought 7 and 6 is 13.

Teacher: You always think differently. [*Friendly laughter from the class*] Thinking 7 and 6 is easier for you?

Nobuhiko: Yes!

Nobuhiko then took the two counters on the right in the top row and moved them down to align them with the counters on the bottom. He then counted them one by one to make sure there were 7 counters on the top and 6 on the bottom.

The teacher then called on Koichi. Koichi is one of the fastest students in the classroom and always seems to have good ideas. He came up to the board, separated counters, and made three groups of 5, 5, and 3.

Koichi: I thought this way: 5 and 5 is 10; so, 3 more is 13.

The teacher then summarized the different approaches shared by students. The making-ten method was called the "Sakiko method," changing 9 + 4 into 7 + 6 was called the "Nobuhiko method," and changing 9 + 4 into 5 + 5 + 3 was called the "Koichi method." The different counting methods were named for the students who shared those particular methods. The Sakiko method (break-apart-to-make-ten method) was the primary method to be taught in this unit according to the curriculum, but the teacher spent approximately equal amounts of time explaining each method at this point by asking students questions.

The teacher then extended his students' exploration of methods by using the new addition situation of "8 + 5" (this requires students to start with a different break-apart pair for 10: 8 and 2). He asked students to apply the methods they had come up with as a class to solve the new problem.

> *Teacher:* Who can explain again how to solve this using the Sakiko method?
>
> *Sanshiro:* We can separate 5 into 2 and 3, and make 10 with 8 and 2, then add 3 to make 13.

The teacher summarized Sanshiro's thinking on the board while verbally reviewing the steps by asking students. After repeating the same process with the other student methods, the teacher asked the students to vote for the method they thought was the easiest to use in this addition situation. Sakiko's method of making-ten received the most votes, and all other methods also got several votes.

In this first lesson of the unit, the teacher welcomed students' different ideas and allowed time for diverse methods. The sharing process was important at the beginning of the unit because it gave an opportunity for students to review previously learned concepts, demonstrate their competency, and set the stage for future exploration. When some students immediately stated the answer for the problem, the teacher recognized their experience and knowledge, yet redirected the students' focus to the process of solving the problem, thus offering opportunities for the students who were experiencing addition with totals in teens for the first time. He then stated the importance of clear explanations to show their thinking with the visual representational supports on the board (see the following section for the discussion of the visual representational supports). When several students shared more basic counting methods, the teacher recognized and valued their contributions even though the focus of the lesson was not officially on counting.

The students' sense of ownership in learning was created and maintained throughout the lesson. The teacher valued the diverse thinking of students by asking them to share their different methods, maintained their ownership of ideas by referring to the methods using the students' names, and encouraged critical thinking by having different students explain and apply the method in different addition situations. Although voting in this initial lesson divided students into different camps, this voting for "the easiest method" continued on to subsequent lessons until all students agreed that "making of ten" was the most useful method. Voting helped individual students express their choices, and with the teacher's gentle guidance, students gradually made their own decisions for their learning.

VISUAL LEARNING SUPPORTS FOR EACH STEP IN THE BREAK-APART-TO-MAKE-TEN METHOD

The teacher used visual representational supports to emphasize the different decomposition steps students had taken in their methods. For the target break-apart-to-make-ten method, he initially supported each step visually. As the unit progressed, he gave support for fewer steps, and at the end only the most difficult step was shown visually. The whole process of the break-apart-to-make-ten method is summarized in figure 7.2.

When the teacher first introduced the visual representational support, he used the extensive Level A questioning (fig. 7.2) to guide students carefully through all four necessary steps while using large counters on the board to assist thinking (fig. 7.1). He effectively combined his questions, the counters on the board, the drawn visual representational support with numbers, and gestures to help students make connections among the different representations and different steps.

The teacher also explained to students that the reason they used the visual representations was to show their thinking in the open. He emphasized that it is important for students to explain their thinking clearly even though he said, "Only answers would be easier." At the beginning of the unit, to draw students' attention to the ten-structured way of seeing the number, the teacher carefully used red chalk to encircle the two numbers that made 10 (steps 3 and 4 of fig. 7.2). Students were also encouraged to use red pencils to circle numbers during the first week in their notebooks. After the first week, the teacher changed to using only white chalk to draw the whole thing. Using the visual representational support to solve problems in their notebooks was very helpful, since it enabled students to keep track of each step of the procedure. Students gradually came to experience the break-apart-to-make-ten method as a fluent whole. From the second week, the teacher allowed each individual student to decide whether he or she would draw the visual representational support or not in his or her notebook. However, whenever he saw students having a problem, he encouraged them to draw the visual representational support to check the thinking procedures.

Students also used smaller versions of the counters the teacher used on the board. For the first three lessons, containers of counters were distributed to each student at the beginning of the class, and students used them to explain their thinking to peers during independent seatwork time. From the fourth class, the teacher kept the containers of counters with him and told students that counters would be available to them if needed. As students became familiar with the visual representational drawings, fewer of them chose to use counters. However, when the teacher saw some students taking a long time or struggling with a certain problem, he suggested that they use the counters to review the process and guide their thinking.

Steps / Levels	1. Realize that 9 needs 1 more to make 10	2. Separate 4 into 1 and 3	3. Add 9 and 1 to make 10	4. Add 10 and 3 to make 13
Level A	"9 and what number make 10?" (Teacher points to 9.)	"What two numbers are you separating 4 into (to make 10)?" (Teacher draws to show break-apart pairs for 4.)	"What do 9 and 1 make?" (Teacher circles 9 and 1 and writes 10 next to the circle.)	"What do 10 and 3 make?" (Teacher points to numbers 10 and 3 and says, "Ten and three" to make connections to "ten-three."
Level B		"What two numbers are you separating 4 into (to make 10)?" (Teacher draws to show break-apart pairs for 4.)	"9 and 1 make …?" (Teacher points to 9 and 1.)	"10 and 3 make …?" (Teacher points to 3.)
Level C		"4 is what number and what number?" (Teacher draws to show break-apart pairs for 4.)		"10 and 3 make …?" (Teacher points to 3.)
Level D		No verbal guiding, teacher uses visual support of drawing break-apart pairs for 4.		

Level E (No guiding of steps)				
Visual representational support in textbooks and on the board	9 + 4 / 1	9 + 4 / 1 3	(9) + 4 / 10 (1) 3	(9) + 4 / **10** (1) **3**

Note: The increasing levels support fewer steps.

Japanese number words, as in other Chinese-based languages, support making ten. They explicitly state "ten" in teen numbers (e.g., "eleven," "twelve," and "thirteen" are said "ten-one," "ten-two," and "ten-three"). Japanese children experience 10 within teen numbers as they hear the "bumps" in the numbers such as "ten-one," "ten-two," and "ten-three." This gives them a clue that after 10, a new sequence starts as the counting in the ones column starts over.

Fig. 7.2. Levels of teacher support during the teens additions unit for learning the break-apart-to-make-ten method

Students also used their fingers as counters. The teacher never discouraged the use of fingers and used them himself sometimes to show students the break-apart pairs of certain numbers. As with counters, students used their fingers less frequently as they came to use the visual representational drawings. The teacher suggested the use of fingers to slower students to confirm break-apart pairs of certain numbers or the amount needed to make ten.

VALUING STUDENTS' DIFFERENT LEARNING PROCESSES

As the unit progressed, students had many opportunities to practice the making-ten process in the classroom: as a whole class orally, as individuals orally in a whole-class situation, as pairs doing independent seatwork, and as individuals doing independent seatwork. Every lesson started with a whole-class review of the break-apart-to-make-ten method, and the teacher's questions guided the students to support their growing competency. The new addition steps were not difficult when they were taken one step at a time, and students developed their understanding of, and fluency with, the overall process as the unit progressed.

All students had opportunities to solve problems individually in the whole-class context supported by questions from the teacher. As individual students gradually came to use the break-apart-to-make-ten method, the different levels of understanding became apparent in the whole-class context. Although the teacher progressed during the unit to supporting fewer steps, when students hesitated or gave wrong answers, he would fall back from a higher level to Levels A, B, or C (see fig. 7.2), supporting each step at that level with questions. Sometimes this "retreat" was used for the whole class, and sometimes it was for individual students.

For example, in the fifth lesson of the unit, the teacher wrote a set of questions on the board that included "9 + another addend" and "8 + another addend" addition problems (fig. 7.3). His initial expectation was that students would state the answers to those questions quickly (Level E support). However, when he saw the first student hesitate, he quickly realized that the students might not be ready for this rapid exercise and changed his expectations. He asked the students to do the hardest step for every problem, finding the needed break-aparts, and he wrote break-apart pairs on the board (Step 2, fig. 7.2).

Teacher: Is this hard? OK, let's do it this way. For 9 + 2, we will separate 2 into what two numbers?

Yutaro: 1 and 1.

When all the break-apart pairs had been written on the board under the second addends, the teacher asked the whole class to say the answer to each question together orally as he pointed to it with a long ruler (Level C support). With the visual break-apart support on the board, students were more confident as they stated the answers together quickly. The teacher then asked individual students to repeat the same procedure in a whole-class context. Most were able to carry out the final two steps on their own, but there were several students who needed additional support. For example, as one student hesitated on her turn, the teacher quickly changed his approach.

Teacher: [*Points to 8 + 4*]

Masayo: 8 + 4 is … 18? 19?

Teacher: Let's slow down and take time here, Masayo [*signaling that it is important to think and that thinking takes time*]. We have 8 + 4. How do we separate 4? (Supporting Step 2)

Masayo: 2 and 2?

Teacher: Then, we have 8 and 2? (Supporting Step 3)

Masayo: 10, so 2 more is 12 (did Step 4 independently).

The whole-class practice context was very important when students shouted out the answers to the teacher's questions together or individually. It offered opportunities for students to share their growing expertise together with their classmates, and it enabled the teacher to assess individual students' progress and assist those who needed it. It was also an enjoyable expe-

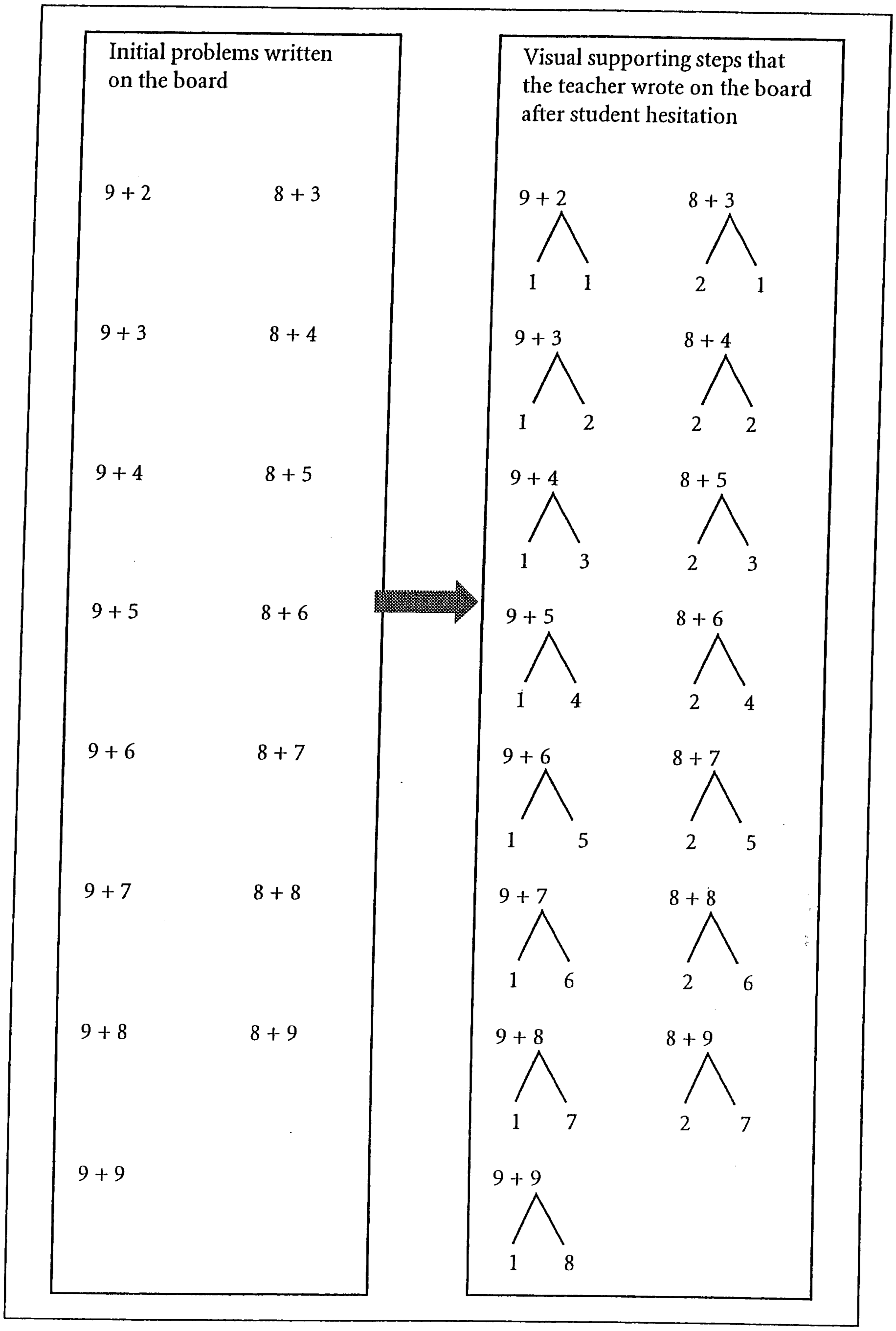

Fig. 7.3. Using "9 + addend" and "8 + addend" problems on the board in Lesson 5

rience where everyone supported one another. Each time an individual student gave an answer to the teacher's question, the student typically asked the whole class, "Is it OK?" When the answer was acceptable, the whole class responded, "It is OK!" to recognize the student's contribution. When the answer was not acceptable, the whole class responded, "It is not OK!" Such a response was typically followed by additional supporting questions from the teacher to trace the steps that the student had taken to arrive at that incorrect answer. This classroom ritual placed students in the role of evaluating one another's mathematical contribution, and the teacher worked to support the evaluation. The feeling of "togetherness" motivated individual students to move forward in the classroom community, and because the slower students were supported as necessary by the teacher, everyone could experience the whole addition process together repeatedly.

This feeling of togetherness was created gradually from the beginning of the school year. Students were given plenty of opportunities to play and work together during the school day both in and outside the classroom. The teacher often supervised student activities indirectly so that students would be responsible for making their own decisions. He also sometimes intentionally created situations where students needed to find ways to work together and solve problems themselves. The teacher believed that his main job was to help students find solutions to their problems, not to give answers. This belief was reflected not only in the way he guided students' learning through questioning but also in the way he supported and guided students' own negotiations in resolving playground conflict. Small-group work was common in the classroom, and cooperation was expected in every aspect of their school lives. Through many successful and not-so-successful experiences of trying to work together, students gradually developed an understanding of one another. They became aware of the strengths and weaknesses of their peers and developed ways to work to support one another. This interdependence among members of the classroom community generated a sense of belonging, and students worked hard to maintain this feeling of togetherness in the community.

Every student was expected to do math well in the classroom. Although the teacher obviously recognized individual developmental differences and adjusted his questions to support the differences, he created a classroom environment that offered ample opportunity for students to experience the concepts and practice the break-apart-to-make-ten addition method together. The existence of different learning paths in the classroom was not viewed as a problem but as a way to help students at different levels. The slower students were involved in the rapid whole-class oral practice through the support of the teacher and thus experienced the whole scope of the process along with the faster, fluent students; the faster students were given time and opportunities to review each step of the method with the slower students.

CONCLUDING REMARKS

The example of the Japanese first-grade classroom presented in this article shows (1) how valuing diverse ideas and learning paths worked to create an effective learning environment in the classroom and (2) how different learning supports helped students learn. Students did not learn the concepts by rote. Instead, in the sense-making environment that supported students' ideas, they remained active agents in their own learning as they followed their own learning paths. The aspects of effective teaching for understanding observed in this situation were also observed in other grades in the school. We summarize them in figure 7.4.

Valuing students' ideas and thinking	• Encourage students to share ideas and different approaches. • Ask students questions to guide their thinking. • Maintain students' ownership of ideas (name different methods with students' names, vote for different methods but discuss advantages and disadvantages).
Providing visual representational support for learning steps	• Use representations (physical objects, drawings, fingers, oral explanations) to strengthen students' understanding. • Make connections among different representations. • Guide students' thinking with visual cues (e.g., circling numbers to make 10, drawing upside-down V to show break-apart pairs) to suggest crucial steps in the process. • Emphasize the critical step of the procedure by using colored conceptual drawing.
Valuing and supporting students' different learning paths	• Vary questioning patterns to meet the different levels of understanding of individual students. • Include slower students in whole-class practice to allow them to experience the entire process rapidly but support them as necessary with questions. • Consider differences among students as strengths, and create situations where they benefit from the differences.

Fig. 7.4. Aspects of teaching for understanding in the Japanese classroom

Teaching is a complex task that helps shape students' perspectives toward mathematics for their future, and students develop their understanding through the experiences provided by their teachers in the classroom. Seeing an example of an effective learning environment that supports students' diverse ideas and learning paths in a Japanese classroom may help us understand how the aspects of teaching that are presented in the NCTM *Standards* work together to create an environment that supports students' engagement, sharing of ideas, and learning. Teachers can help all students learn both mathematically and culturally valuable methods in a meaningful and sensitive way adapted to students' individual learning paths.

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