

# Mathematics Tools Used in Mathematics K'UbX]b'gh[ fUXY



Counters



Counting blocks



Ten-frame dot cards



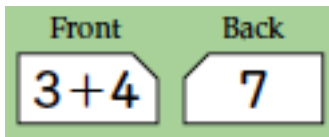
Numeral cards



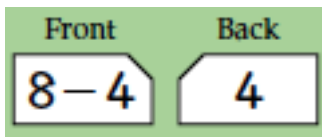
Counting blocks and 10-block tray



Counting blocks, 10-block tray, and sleeve



Addition cards

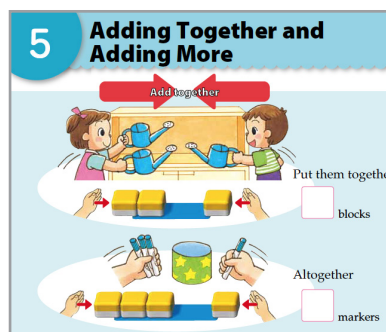


Subtraction cards

## Development and Connection of Increasingly Abstract Representations of Mathematics

The mathematics tools shown above help students learn to model with mathematics (MP4), as the following examples illustrate.

- ◆ “Real world” and pictured events provide contexts for students to interpret, quantify and solve problems using the language of mathematics. For example, in the picture from Unit 1 at the right, students count with one-to-one correspondence and group characters and objects within the pictured event.
- ◆ Students use concrete manipulatives to represent numeric relationships and operations. In Unit 1 lessons, students place counting cubes directly on top of the animals, making a one-to-one correspondence. The cubes can then be removed and counted to find the cardinal number that tells how large the set is. Students are reasoning back and forth between story context and abstract counting to quantify (MP2).
- ◆ Pictures, diagrams, and ten frame dot cards help students develop visual images that facilitate mental math. For example, the 10-frame dot cards in Unit 1 (p. 15) represent the quantities 3, 4, and 5. The dot cards are strategically placed under the picture of the situation. The ten frame’s semi-concrete representation translates the (concrete) animals into a sequence of dots. The dots are matched to the (abstract) numerals representing cardinal numbers. As this model of connecting representations is repeated for numbers 6 through 10, students learn to look for and make use of structure and to connect the concrete, semi-concrete, and abstract representations (MP7).



- ◆ Written symbols include numerals, math sentences and abstract symbols. For kindergarten students important milestones include writing numerals, beginning to understand the place value of the base ten number system, and learning the conventions of writing math sentences (expressions and equations).

As the preceding examples illustrate, concrete and semi-concrete objects (and the actions on these objects) are represented ultimately by the abstract symbol system of numerals and written expressions or equations. Although it is easier for students to understand a concept if they first manipulate objects, a primary goal of mathematics is to learn to think quantitatively and abstractly (MP2). Students take the essential elements of an experience and represent them abstractly in order to think about and solve mathematical problems. The concrete, semi-concrete, and abstract approach described here is critical to advancing student learning. Strategies for using these materials become second nature to students as they engage in lessons, study textbook pictures, and participate in ongoing discussion and practice.

## *g; fUXY. Development and Connection of Increasingly Abstract Representations of Mathematics*

The mathematics tools shown above help students learn to model with mathematics (MP4), as the following examples illustrate.

- ◆ “Real world” and pictured events provide contexts for students to interpret, quantify and solve problems using the language of mathematics. For example, in the picture from Unit 8 at the right, seven students are about to eat cake from a table with 9 pieces of cake. Students make one-to-one correspondence between two different types of quantities within the pictured context.
- ◆ Pictures, diagrams, tables, graphs and tape diagrams help students develop visual images that facilitate mental math and quantitative reasoning. To the right, the triangles and circles representing pieces of cake and students, respectively, are strategically placed under the picture of the situation. The diagram helps students understand the structure of the problem, identify what is known and unknown, and determine the appropriate operation. Students use the representations to communicate their ideas and thinking processes and to justify their solution methods. (MP7) .

**4** There are 9 pieces of cake.  
7 people each eat 1 piece.  
How many pieces are there left over?

Cake ( ) pieces

People ( ) pieces

( ) people

Match the ▲ and ● with a line.

( ) pieces will be eaten.

Where in the chart is the answer?

Math Science

Answer ( ) pieces

57

- ◆ Written symbols include numerals, math sentences and abstract symbols. Important milestones for Grade 1 students include writing numerals to 120, understanding the place value of the base ten number system and learning the conventions of writing math sentences (expressions and equations).

Concrete and semi-concrete objects (and the actions on these objects) are ultimately represented using the abstract symbol system of numerals and written expressions or equations. For example, in Units 5 and 6, students use the counting blocks and 10-block trays to think about the process of calculation with regrouping. Although it is easier for students to understand a concept if they first manipulate objects, a primary goal of mathematics is to learn to think quantitatively and abstractly (MP2). Students take the essential elements of an experience and represent them abstractly in order to think about and solve mathematical problems. Connecting the concrete, semi-concrete, and abstract approaches described here is critical to advancing student learning. Strategies for using the materials become second nature to students as they engage in lessons, study textbook pictures, and participate in ongoing discussion and practice.