Appendix D: Properties of Operations, Equality and Inequality

Properties of Operations

The table below illustrates the properties of operations. For each property, the variables a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system and the complex number system.

Property of Operation	**Example**
Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of zero	$a + 0 = a$ $0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = 0$ and $(-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of one	$a \times 1 = a$ $1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = 1$ and $\frac{1}{a} \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = (a \times b) + (a \times c)$

Properties of Equality

The table below illustrates the properties of equality. For each property, the variables a, b and c stand for arbitrary numbers in a given number system. The properties of equality apply to the rational number system, the real number system and the complex number system.

Property of Equality	**Example**
Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Properties of Inequality

The table below illustrates the properties of inequality. For each property, the variables a, b and c stand for arbitrary numbers in a given number system. In addition, exactly one of the following is true: $a < b$, $a = b$ or $a > b$. The properties of inequality apply to the rational number system and the real number system.

Property of Inequality	**Example**
Asymmetric property of inequality	If $a > b$, then $b < a$.
Transitive property of inequality	If $a > b$ and $b > c$, then $a > c$.
Addition property of inequality	If $a > b$, then $a + c > b + c$.
Subtraction property of inequality	If $a > b$, then $a - c > b - c$.
Multiplication property of inequality	If $a > b$ and $c > 0$, then $a \times c > b \times c$. If $a > b$ and $c < 0$, then $a \times c < b \times c$.
Division property of inequality	If $a > b$ and $c > 0$, then $a \div c > b \div c$. If $a > b$ and $c < 0$, then $a \div c < b \div c$.